Some Questions for Chapter 6

1. Can you work each homework and quiz problem *correctly* and *quickly*, providing explanations and justifications, without looking at the textbook?
2. Have you carefully studied the material in Sections 6.1–6.7?
3. Have you carefully studied the slides on the website http://www.ms.uky.edu/~lee/ma111fa09/ma111fa09.html?
4. What is a Hamilton path?
5. What is a Hamilton circuit?
6. What is a complete graph?
7. What is K_N?
8. What is the formula for the number of edges of K_N and where does this formula come from?
9. What is the formula for the number of Hamilton circuits in K_N and where does this formula come from?
10. What is the meaning of $m!$ (factorial)?
11. What is a Traveling Salesman Problem?
12. Can you carry out the Brute-Force Algorithm?
13. Can you carry out the Nearest-Neighbor Algorithm?
14. Can you carry out the Repetitive Nearest-Neighbor Algorithm?
15. What algorithm must you use if you are asked to find an optimal tour?
16. How can you calculate the relative error of a tour as compared to the optimal tour?

MA 111 Review for Exam 2

Exam 2 (given in class on Monday, Sept. 28) will cover Unit 2: The Mathematics of Touring.

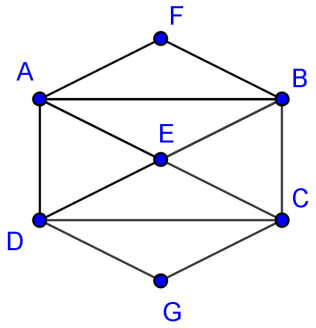
You should be familiar with the following key ideas:

1. Know the definitions of Hamilton path and Hamilton circuit. Be able to say why a given path/circuit is or is not a Hamilton path/circuit.
2. Understand the definition of a complete graph K_N. Be able to compute the number of edges and the number of different Hamilton circuits in K_N for any N.
3. Know the definition of a weighted graph.
4. Understand the goal of solving a traveling salesman problem.
5. Be able to implement the Brute Force Algorithm, the Nearest Neighbor Algorithm, and the Repetitive Nearest Neighbor Algorithm, for finding Hamilton circuits in complete graphs.
6. Understand the pros and cons of using any of the algorithms above.
7. Be able to rewrite a given Hamilton circuit so that it starts and ends at a designated reference point.

Practice Problems:

Use exercises in the text to supplement these for extra practice. (The odd-numbered problems have solutions in the back of the book so that you can check your answers.) Review the homework exercises and the examples in the text.

1. Consider the following graph:



Which of the following lists of vertices describe a Hamilton circuit? For those that do not describe Hamilton circuits, why not?

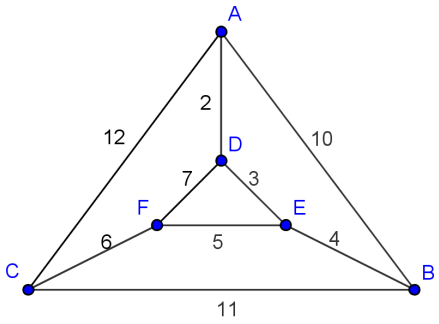
(a) A, F, B, C, G, D

(b) A, F, B, C, G, D, A

(c) B, E, A, F, D, G, C, B

(d) B, F, A, E, D, G, C, B

2. For the following weighted graph:



(a) Find the total weight of the path: C, F, D, A, B, E

(b) Find a Hamilton path that starts at A and ends at C, and give its total weight.

3. (a) How many edges are in K_{15}, the complete graph with 15 vertices?

   (b) How many distinct Hamilton circuits are contained in K_8?

   (c) How many distinct Hamilton circuits are contained in K_N?

4. You work at FedEx, and the company has just decided to add a new truck route that will cover 9 cities in China. (You can assume that each city is connected to each other city.)

   You want to design the route in the most efficient way, so that the truck will never have to visit a city more than once, except for Beijing, where the FedEx headquarters are. Also for efficiency, you want the total distance traveled to be as small as possible.

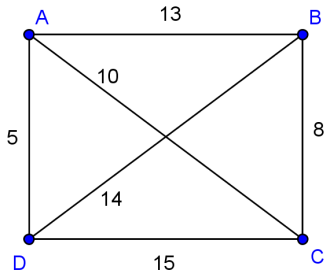
   Your boss wants to know the *optimal* route.

   (a) Which algorithm should you use to answer your boss's question?

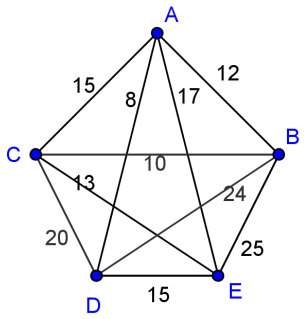
   (b) Suppose that for one possible route, your computer can compute the total distance traveled in half of a second. Using the algorithm you named in part a, how many hours will you need to find the optimal tour?

   (*Hint: We are considering the weighted graph K_9, where the weights are given by the distances between cities.*

5. For the following weighted graph, use the Brute Force Algorithm to find an optimal Hamilton circuit starting at vertex C.



6. Consider the following weighted graph:

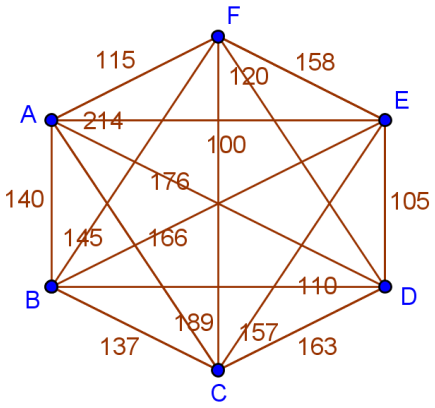


(a) Suppose the tour must start at vertex A and then go directly to vertex B. Use the Brute Force Algorithm to find an optimal Hamilton circuit under these conditions.

(b) Use the Nearest Neighbor Algorithm to find the nearest neighbor tour starting at vertex A.

(c) How could you write the circuit you just found in part b so that it starts at vertex D?

7. Consider the following graph:



(a) Apply the Nearest Neighbor Algorithm, starting at vertex B.

(b) Now apply the Nearest Neighbor Algorithm, starting at vertex F.

(c) Now implement the Repetitive Nearest Neighbor Algorithm.

8. Suppose you are planning a trip around the world. Below is a table of the distances (in miles) between the cities you want to visit. Assume that you will start and end your trip in New York City.

	New York	London	Shanghai	Istanbul	Moscow	Buenos Aires
New York		3470	7400	5030	4700	5250
London	3470		5715	1555	1560	6920
Shanghai	7400	5715		4960	4235	12200
Istanbul	5030	1555	4960		1090	7600
Moscow	4700	1560	4235	1090		8400
Buenos Aires	5250	6920	12200	7600	8400	

Use the Nearest Neighbor Algorithm to find a near-optimal route for your trip. What is the total distance traveled?