MA 111 Agenda - Monday, 11.28.2011

Notes from Elizabeth Weaver

11.28.1 The Lone-Divider Method

- The divider-chooser method only works when we are trying to divide a continuous set of goods S between two people. How can we divide up goods between more than two people?
- In sections 3.3 and 3.4 we will look at ways to extend the divider-chooser method to N players. The first extension we will look at is called the lone-divider method.
- The lone-divider method can be used when our set of goods to be divided is continuous. For the sake of ease, throughout this section we will refer to S as a cake.
- The more players you have, the more complicated the situation can become. We will first concentrate on the situation with three players.
- **The Lone-Divider Method for Three Players:**
  1. **Preliminaries:** One of the players will be the divider D, and the other two players will be the choosers, C_1 and C_2. Since it is better to be a chooser, this can be decided by a random draw.
  2. **Division:** The divider, D, divides the cake into three shares, s_1, s_2, s_3. Since D does not know what share he will get, he is forced to divide the cake into three shares of equal value in his opinion.
  3. **Bidding:** Each chooser independently writes up a bid listing which of the pieces are fair shares in his opinion. A chooser's bid *must* contain every single piece that he considers to be a fair share. To maintain our privacy requirement, it is important that the choosers write their bids independently.
  4. **Distribution:** We will separate the pieces into two types: C-pieces (pieces chosen by at least one of the choosers) and U-pieces (pieces that did not appear on either of the choosers bids). Our outcome will depend on the number of C-pieces present.
     - **Case 1:** When there are two or more C-pieces, there is always a way to give each chooser a different piece from among the pieces listed in his bid. The divider then gets the remaining piece.
     - **Case 2:** When there is only one C-piece, we have a problem. In this situation, both of the choosers are bidding for the same piece, and we cannot give it to both of them. We first give one of the unwanted pieces to D. We then combine the two remaining pieces into one piece and have the two players divide it using the divider-chooser method.

11.28.2 Examples

- **Example:** Diane, Cliff, and Carla are dividing a cake using the lone-divider method. Diane is the divider, and Cliff and Carla are the choosers. Diane divides the cake into three pieces s_1, s_2, and s_3. The table below shows the values of the pieces in the opinion of each of the players. Determine each chooser's bid, and find a fair division.

	s_1	s_2	s_3
Diane	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$
Cliff	35%	10%	55%
Carla	40%	25%	35%

- **Example:** As above, Diane, Cliff, and Carla are dividing a cake using the lone-divider method. Diane is the divider, and Cliff and Carla are the choosers. Determine each chooser's bid, and find a fair division.

	s_1	s_2	s_3
Diane	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$
Cliff	30%	40%	30%
Carla	60%	15%	25%

- **Example:** As above, Diane, Cliff, and Carla are dividing a cake using the lone-divider method. Diane is the divider, and Cliff and Carla are the choosers. Determine each chooser's bid, and find a fair division.

	s_1	s_2	s_3
Diane	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$
Cliff	20%	30%	50%
Carla	10%	20%	70%

11.28.3 The Lone-Divider Method for More Than Three Players

- The lone-divider method for more than three players is very similar to the method with three players. The main difference will occur in the distribution step.
- **The Lone-Divider Method for More Than Three Players**
  1. **Preliminaries:** One of the players will be the divider D, and the other $N-1$ players will be the choosers, C_1,...,C_{N-1}. Since it is better to be a chooser, this can be decided by a random draw.
  2. **Division:** The divider, D, divides the cake into N equal shares, $s_1, ..., s_N$.
  3. **Bidding:** Each chooser independently writes up a bid listing which of the pieces are fair shares in his opinion.
  4. **Distribution:**
     - **Case 1:** If there is a way to assign a different share to each of the $N-1$ choosers based on their bids, then that should be done. The remaining share is assigned to the divider.
     - **Case 2:** There could be a standoff. This case is more complicated, but you can read about it on page 85.
- **Example:** Suppose Desmond, Claire, Charlie, and Charlotte are dividing a cake using the lone-divider method where Desmond is the divider. The table below shows how each of the players values each of the four shares. Fill in the missing values, and find a fair-division, if possible.

	s_1	s_2	s_3	s_4
Desmond				
Claire	30%	20%	35%	
Charlie		20%	40%	20%
Charlotte	25%	20%		35%

- Five players are dividing a cake among themselves using the lone-divider method. After the divider D cuts the cake into five slices $(s_1, s_2, s_3, s_4, s_5)$, the choosers C_1, C_2, C_3, and C_4 submit their bids for these shares. Suppose that the chooser's bids are $C_1 : \{s_2, s_4\}$, $C_2 : \{s_2, s_4\}$, $C_3 : \{s_2, s_3, s_5\}$, and $C_4 : \{s_2, s_3, s_4\}$. Describe two different fair divisions of the cake. Explain why these are the only two fair divisions of the cake.
- Four players (Angela, Booth, Brennan and Hodgins) are sharing a cake which they jointly purchased for $20. The divider splits the cake into four slices (s_1, s_2, s_3, s_4). The following table gives the values of some of the slices to each of the players.

	s_1	s_2	s_3	s_4
Angela	$3.00	$5.00	$5.00	
Booth	$4.50	$6.75	$5.25	
Brennan	$5.00	$5.00	$5.00	
Hodgins	$4.75	$4.75	$4.75	

  1. Fill in the missing values for each of the players.
  2. Which of the four players was the divider?
  3. Determine the bid for each of the choosers.
  4. Find a fair division of the cake using s_1, s_2, s_3 and s_4 as fair shares.

11.28.4 After this lecture, you should be able to...

- explain the steps in the lone-divider method.
- determine players bids based on the values they give the shares.
- find a fair-division using the lone-divider method when given enough information.
- work on all of the problems in Homework 12.