MA 111 Notes on Lone-Divider: The Case of a Conflict
December 2, 2011

Jon, Ron, and Don are going to split a sub sandwich. The sandwich is 9 inches long. The first three inches is roast beef, the second three inches is turkey, and the last three inches is vegetarian. You are given

- Jon is indifferent. He likes everything equally.
- Ron likes turkey and vegetarian equally, and he likes roast beef twice as much as he likes turkey.
- Don likes roast beef three times more than he likes turkey and he doesn't like vegetarian at all.

They will divide the sandwich using the lone divider method.

Suppose Jon is selected to play the role of the divider.

(a) How will Jon divide the sandwich?

(b) How does Ron value the three slices, and which slices will Ron include in his bidset?

(c) How does Don value the three slices, and which slices will Don include in his bidset?

There is a conflict! Ron and Don each are fighting for the roast beef slice. How do we resolve this conflict? Jon will be given one of the unwanted slices (the so-called U pieces). Let's assume Jon receives the vegetarian slice. Ron and Don will now enter into a fair division game to divide the 6 inch, roast beef and turkey sub. Now, all fair division methods reduce to the two person "I cut-you choose" method. Suppose Don is selected to be the divider. (At this point, the vegetarian slice is gone; Don and Ron are each trying to get at least 50% of the six inch sub. At the end of the problem, we will see how much their slices are worth in terms of the original sub.)

(d) How will Don divide the sandwich?

(e) How does Ron value each of the two slices, and which slice will Ron choose?

Jon, Ron, and Don now each has a slice of the original 9 inch sub. Did they each get at least one third of the value of the whole sub?

(f) Jon received the vegetarian slice, which Jon thinks is 1/3 of the value of the whole sub.

(g) How much is Don's slice worth (as a percentage or fraction of the 9 inch sub)?

(h) How much is Ron's slice worth (as a percentage or fraction of the 9 inch sub)?

Lone-Chooser: Another Fair Division Method

Jon, Ron, and Don could have used another natural method which would guarantee each player a fair share of the original sub. In this method, one of the players will serve as the chooser, and the remaining players will act as dividers. Suppose Don is selected to be the chooser, so Jon and Ron will be dividers.

Round 1: Jon and Ron will divide the 9 inch sandwich between themselves. They will do this using the two person "I cut-you choose" method.

(a) Suppose Ron is selected to be the divider. How will Ron cut the sub?

(b) How does Jon value each of the two slices? Which slice will Jon choose?

Now, Ron and Jon each have a slice of the big sub. Call their slices R and J.

Round 2: Ron and Jon each have at least 50% of the value of the sandwich, while poor Don has nothing. Ron and Jon each will subdivide their slice into three fair slices. Then Don will take one of Ron's slices and one of Jon's slices.

(c) How will Jon subdivide his slice?

(d) How does Don value each of these three slices? Which one will Don take?

(e) How will Ron subdivide his slice?

(f) How does Don value each of these three slices? Which one will Don take?

Final analysis: Is it fair?
  (g) How much are Jon's two slices worth?

(h) How much are Don's two slices worth?

(i) How much are Ron's two slices worth?