MA111 — Chapter 5 Review

Exam 3 on Chapter 5 will be given in class on Monday, October 24. Can you work each homework and quiz problem correctly and quickly, providing explanations and justifications, without looking at the notes? Have you carefully studied the slides on the website? You should be familiar with the following key ideas:

1. Know what a graph is and what its vertices and edges are.
2. Understand that a graph can be represented by a drawing (in which, possibly, the edges cross), but also by the list of its vertices and the list of its edges. Understand that there may be many different drawings for the same graph.
3. Be able to create the list of vertices and edges from the drawing of a graph, and to create a drawing of a graph from the list of vertices and edges.
4. Know what loops, multiple edges, and isolated vertices are, and how to identify them from the drawing of a graph as well as from the list of vertices and edges.
5. Know what it means for two vertices to be adjacent — they are directly joined by an edge. Vertices with loops on them are adjacent to themselves.
6. Know what it means for two edges to be adjacent — the two edges share a common vertex.
7. Be able to determine the degrees of vertices. Be able to identify even and odd vertices.
8. Given a list of vertices with specified degrees, be able to draw a graph with vertices having these degrees.
9. Know what a path is (the starting and ending vertices are different) and what a circuit is (the starting and ending vertices are the same). Know that in paths and circuits, edges can be traveled no more than once. Be able to identify and count paths and circuits in a graph. Be able to determine the length of a path or a circuit. Know that a single loop is a circuit of length 1.
10. Know what it means for a graph to be connected or disconnected. Know what the components of a graph are, and how to identify them. Know that an isolated vertex is a component in and of itself.
11. Know what it means for an edge of a graph to be a bridge, and how to identify bridges in a graph.

12. Know what an Euler path is (a path using every edge exactly one) and what an Euler circuit is (a circuit using every edge exactly once).

13. Know what it means for a drawing to have a unicursal tracing — the figure can be drawn with either an Euler path or an Euler circuit.

14. Know that a given graph cannot have both an Euler path and an Euler circuit — it can have one or the other or neither. Be able to recognize Euler paths and circuits.

15. Know how to use Euler's Theorems to recognize whether or not a graph has an Euler path or an Euler circuit. It has an Euler circuit if and only if it is connected and all vertices are even. It has an Euler path if and only if it is connected and there are exactly two odd vertices (and in this case the Euler path must start at one odd vertex and end at the other). If a graph has more than two odd vertices, then it has no Euler path or Euler circuit.

16. Know how to find Euler paths and circuits in a graph. First identify where you can start and end. Then, as you trace the edges, do not trace any edge that leaves portions of untraced edges or the final vertex inaccessible. In other words, don't use a bridge in the graph of untraced edges unless you must. (This is known as Fleury's algorithm.)

17. Be able to use graphs to model real-life problems, such as street systems in which you may need to travel on certain streets once or twice, or systems of land masses and bridges in which you may need to cross certain bridges once or twice.

18. Be able to explain why the sum of the degrees of all the vertices equals twice the number of edges, and therefore why a graph always has an even number of odd vertices.

19. Know what exhaustive routes are–a route that travels passes through each and every edge at least once.

20. Know how to Eulerize a graph—how to duplicate certain edges in order to make every vertex even. Be able to use this to find optimal exhaustive closed routes (starting and ending at the same vertex) by duplicating as few edges as possible.

21. Know how to semi-Eulerize a graph—how to duplicate certain edges in order to leave exactly two odd vertices. Be able to use this to find optimal exhaustive open routes (starting and ending at different vertices) by duplicating as few edges as possible.