MA 111 Review for Exam 2

Exam 2 will be given in class on Monday, October 3. I will provide a formula sheet. You will need to know which formula to use for each problem.

Can you work each homework and quiz problem *correctly* and *quickly*, providing explanations and justifications, without looking at the notes?

Have you carefully studied the slides on the website?

You should be familiar with the following key ideas:

- Understand what percent means..
- Be able to find different variables in a percentage calculation (e.g., use the base and the result to find the percent).
- Understand how changes in numbers can be measured by percents (percent change).
- Know how to increase or decrease a number by a percentage.
- Understand simple interest. Be able to solve for any of the variables in the simple interest formula if given the other three. Here is the simple interest formula:

$$F = P(1 + rt).$$

- Understand the concept of compound interest. Be able to use the compound interest formula to solve for the variable P or F. Here is the compound interest formula:

$$F = P\left(1 + \frac{r}{n}\right)^{nt}$$

or

$$F = P\left(1 + p\right)^{T}.$$

And here is the compound interest formula if interest is compounded continuously:

$$F = Pe^{rt}.$$

- Know the definition of APY and be able to find the APY for a given investment. Be able to use APY to compare investments. To find the APY, just check the percent growth of \$100 in one year.
- Understand what an annuity is, and be able to distinguish between a deferred annuity and an installment loan.

- Know how to use the deferred annuity formula:

$$F = L\left[\frac{(1+p)^T - 1}{p}\right].$$

  Use $L = P(1+p)$ if the last payment is at the beginning of the last pay period (which is usually the case), but $L = P$ if the last payment is at the end of the last pay period. Be able to use the formula to solve for F or P.

- Know how to use the amortization formula:

$$P = Fq\left[\frac{q^T - 1}{q - 1}\right],$$

  where $q = \frac{1}{1+p}$. It is usually the case that the first payment is at the end of the first pay period. Be able to solve for F or P. If it happens that the first payment is at the beginning of the first pay period, use the formula

$$P = F\left[\frac{q^T - 1}{q - 1}\right].$$

- Be able to compare different financing options in the context of buying, say, a car or house.

Practice Problems

1. Express each of the following percentages as a decimal.
   (a) 43%
   (b) 1.5%
   (c) 250%
   (d) 0.13%
2. If your score on a test was 62 points out of 70, express the score as a percentage.
3. What is 37% of 56?
4. 123 is 17% of what?
5. If your income in one year is \$65000 and you pay \$10000 in income tax, what is the tax rate?
6. If you buy a book that costs \$19.95, and the sales tax is 6%, what amount do you owe?
7. In 2000, tuition at UK was \$6550. In 2001, tuition increased by 7%. How much was tuition at UK in 2001?
8. A computer that usually sells for \$830 is on sale for 15% off. How much does the computer cost?
9. One year ago, a gallon of gas sold for \$2.82. Now the price of a gallon of gas is \$2.63. By what percent has the price decreased over the past year?
10. The population of Pleasantville this year is 5,900. This is a 3% increase over last year's population. What was the size of the population of Pleasantville last year?
11. (a) If 300 is decreased by 15% and then the result is increased by 10%, what is the final result?
    (b) If the price of gas increases by 7%, and then increases by 5%, and then decreases by 10%, by what percent has the price of gas changed?
12. Your parents purchased a government bond 18 years ago at $5\frac{1}{2}\%$ annual simple interest. Now it is worth \$6965. How much did your parents pay for the bond?
13. At what annual simple interest rate would an investment triple in 50 years?

14. If you invest \$1000 at a simple annual interest rate of 4.5%, how many years will it take for this amount to grow to \$1270?

15. Suppose you deposit \$250 into a savings account with a 5% APR.

    (a) If the interest is compounded yearly, how much money will be in the account in 20 years?

    (b) If the interest is compounded daily, how much money will be in the account in 20 years?

    (c) If the interest is compounded continuously, how much money will be in the account in 20 years?

16. Use the APY to answer the following: Which of the following investments is better: a 4% APR compounded monthly, a 4.25% APR compounded annually, or a 3.8% APR compounded continously?

17. A savings account at your bank offers a 4% APR compounded weekly.

    (a) If you make only one deposit of \$1500, and then don't touch the account, how much money will be in the account in 3 years?

    (b) If instead you deposit \$15 each week, how much money will be in the account in 3 years? (Assume that you deposit money at the beginning of the week, and interest is added at the end of the week.)

18. You would like to save \$7,000 for an overseas trip that will begin in 18 months. If you plan to make a daily deposit into a savings account with an APR of 7%, compounded daily, how much should your daily deposit be in order to meet your goal? Assume there are exactly 30 days in each month, and that the last deposit generates no interest.

19. (a) Suppose you wish to purchase a car that costs \$11,500. If you can get a 10 year loan with a 5% APR compounded monthly, how much will your monthly payment be?

    (b) With such monthly payments, how much interest will you pay over the 10 years?

20. Your bank approves you for a 30 year home loan with a 6% APR compounded monthly and monthly payments of \$800. What price can you afford for your new house?

Solutions

1. Express each of the following percentages as a decimal.

   (a) 43%. 0.43

   (b) 1.5%. 0.015

   (c) 250% 2.50

   (d) 0.13% 0.0013

2. If your score on a test was 62 points out of 70, express the score as a percentage.

   $\frac{62}{70} \times 100 = 88.57\%$.

3. What is 37% of 56?

   $0.37 \times 56 = 20.72$.

4. 123 is 17% of what?

   $0.17A = 123$, so $A = 123/0.17 = 723.53$.

5. If your income in one year is \$65000 and you pay \$10000 in income tax, what is the tax rate?

   $\frac{10000}{65000} \times 100 = 15.38\%$.

6. If you buy a book that costs \$19.95, and the sales tax is 6%, what amount do you owe?

   $0.06 \times 19.95 = \$1.20$.

7. In 2000, tuition at UK was \$6550. In 2001, tuition increased by 7%. How much was tuition at UK in 2001?

   $6550(1 + 0.07) = \$7008.50$.

8. A computer that usually sells for \$830 is on sale for 15% off. How much does the computer cost?

   $830(1 - 0.15) = \$705.50$.

9. One year ago, a gallon of gas sold for \$2.82. Now the price of a gallon of gas is \$2.63. By what percent has the price decreased over the past year?

   $\frac{2.63-2.82}{2.82} \times 100 = -6.74\%$.

10. The population of Pleasantville this year is 5,900. This is a 3% increase over last year's population. What was the size of the population of Pleasantville last year?

    $5900 = A(1 + 0.03)$, so $A = 5728$.

11. (a) If 300 is decreased by 15% and then the result is increased by 10%, what is the final result?

    $300(1 - 0.15)(1 + 0.10) = \$280.50$.

    (b) If the price of gas increases by 7%, and then increases by 5%, and then decreases by 10%, by what percent has the price of gas changed?

    Call the original price A. The new price is $A(1 + 0.07)(1 + 0.05)(1 - 0.10) = A(1.01115)$. This is equal to $A(1 + 0.01115)$, so the price has increased by about 1.11%.

12. Your parents purchased a government bond 18 years ago at $5\frac{1}{2}\%$ annual simple interest. Now it is worth \$6965. How much did your parents pay for the bond?

    Use $F = P(1 + rt)$. Here, $R = 5.5\%$, $r = \frac{5.5}{100} = 0.055$, $t = 18$, $F = 6965$. So $6965 = P(1 + 0.055 \cdot 18) = P(1.99)$; $P = \frac{6965}{1.99} = \$3500$.

13. At what annual simple interest rate would an investment triple in 50 years?

    For convenience, use \$100, which must grow to \$300. Use $F = P(1+rt)$. Here, $t = 50$. So $300 = 100(1 + 50r)$, so $3 = 1 + 50r$, $2 = 50r$, $r = 0.04$. So the interest rate is 4%.

14. If you invest \$1000 at a simple annual interest rate of 4.5%, how many years will it take for this amount to grow to \$1270?

    Use $F = P(1 + rt)$. Here, $P = 1000$, $F = 1270$, $R = 4.5\%$, $r = 0.045$. So $1270 = 1000(1 + 0.045t)$. Solving for t, $1.27 = 1 + 0.045t$, $0.27 = 0.045t$, $t = 6$.

15. Suppose you deposit \$250 into a savings account with a 5% APR.

    (a) If the interest is compounded yearly, how much money will be in the account in 20 years?

    Use $F = P(1 + \frac{r}{n})^{nt}$. Here, $R = 5\%$, $r = \frac{5}{100} = 0.05$, $n = 1$, $P = 250$, $t = 20$. So $F = 250(1 + \frac{0.05}{1})^{1 \cdot 20} = \$663.32$.

    (b) If the interest is compounded daily, how much money will be in the account in 20 years?

    Use $F = P(1 + \frac{r}{n})^{nt}$. Here, $R = 5\%$, $r = \frac{5}{100} = 0.05$, $n = 365$, $P = 250$, $t = 20$. So $F = 250(1 + \frac{0.05}{365})^{365 \cdot 20} = \$679.52$.

(c) If the interest is compounded continuously, how much money will be in the account in 20 years?

Use $F = Pe^{rt}$. Here, $R = 5\%$, $r = 0.05$, $P = 250$, $t = 20$. So $F = 250e^{(0.05)(20)} = \$679.57$.

16. Use the APY to answer the following: Which of the following investments is better: a 4% APR compounded monthly, a 4.25% APR compounded annually, or a 3.8% APR compounded continuously?

Check to see by what percent \$100 grows in one year in each case. Use $F = P(1 + \frac{r}{n})^{nt}$. In the first case, $R = 4$, $r = \frac{4}{100} = 0.04$, $n = 12$, $P = 100$, $t = 1$. So $F = 100(1 + \frac{0.04}{12})^{12} = \$104.07$. This means that the APY is $\frac{104.07-100}{100} \times 100 = 4.07\%$. In the second case, $R = 4.25$, $r = \frac{4.25}{100} = 0.0425$, $n = 1$, $P = 100$, $t = 1$. So $F = 100(1 + \frac{0.0425}{1})^1 = \$104.25$. This means that the APY is $\frac{104.25-100}{100} \times 100 = 4.25\%$. In the third case, $R = 3.8$, $r = 0.038$, $P = 100$, $t = 1$. So $F = 100e^{(0.038)(1)} = \$103.87$. This means that the APY is $\frac{103.87-100}{100} \times 100 = 3.87\%$. So the second option is better.

So the second investment is better.

17. A savings account at your bank offers a 4% APR compounded weekly.

(a) If you make only one deposit of \$1500, and then don't touch the account, how much money will be in the account in 3 years?

Use $F = P(1 + \frac{r}{n})^{nt}$. Here, $R = 4\%$, $r = \frac{4}{100} = 0.04$, $n = 52$, $P = 1500$, $t = 3$. So $F = 1500(1 + \frac{0.04}{52})^{52 \cdot 3} = \$1691.17$.

(b) If instead you deposit \$15 each week, how much money will be in the account in 3 years? (Assume that you deposit money at the beginning of the week, and interest is added at the end of the week.)

Use $F = L\left[\frac{(1+p)^T - 1}{p}\right]$. Here, $R = 4$, $r = \frac{4}{100} = 0.04$, $n = 52$, $p = \frac{0.04}{52}$, $P = 15$, $L = 15(1 + p) = 15(1 + \frac{0.04}{52})$, $T = 52 \cdot 3 = 156$. So

$$\begin{aligned} F &= 15\left(1 + \tfrac{0.04}{52}\right)\left[\frac{(1+\frac{0.04}{52})^{156} - 1}{\frac{0.04}{52}}\right] \\ &= \$2487.09. \end{aligned}$$

18. You would like to save \$7,000 for an overseas trip that will begin in 18 months. If you plan to make a daily deposit into a savings account with an APR of 7%, compounded daily, how much should your daily deposit be in order to meet your goal? Assume there are exactly 30 days in each month, and that the last deposit generates no interest.

Use $F = L\left[\frac{(1+p)^T-1}{p}\right]$. Here, $R = 7\%$, $r = \frac{7}{100} = 0.07$, $n = 365$, $p = \frac{0.07}{365}$, $F = 7000$, $T = 18 \cdot 30 = 540$, $L = P$ (because the last deposit generates no interest). So

$$\begin{array}{rcl} 7000 & = & P\left[\frac{(1+\frac{0.07}{365})^{540}-1}{\frac{0.07}{365}}\right] \\ 7000 & = & P(568.895) \\ \frac{7000}{568.895} & = & P \\ \$12.30 & = & P. \end{array}$$

19. (a) Suppose you wish to purchase a car that costs \$11,500. If you can get a 10 year loan with a 5% APR compounded monthly, how much will your monthly payment be?

Use $P = Fq\left[\frac{q^T-1}{q-1}\right]$. Here, $R = 5\%$, $r = \frac{5}{100} = 0.05$, $n = 12$, $p = \frac{0.05}{12}$, $q = \frac{1}{1+\frac{0.05}{12}} = 0.995851$, $P = 11,500$, $T = 10 \cdot 12 = 120$. So

$$\begin{array}{rcl} 11500 & = & F(0.995851)\left[\frac{0.995851^{120}-1}{0.995851-1}\right] \\ 11500 & = & F(94.2833) \\ \frac{11500}{94.2833} & = & \$121.97 \end{array}$$

(b) With such monthly payments, how much interest will you pay over the 10 years?

Total of all payments: $121.97 \times 120 = \$14636.40$. Total interest paid: $14636.40 - 11500 = \$3136.40$.

20. Your bank approves you for a 30 year home loan with a 6% APR compounded monthly and monthly payments of \$800. What price can you afford for your new house?

Use $P = Fq\left[\frac{q^T-1}{q-1}\right]$. Here, $R = 6\%$, $r = \frac{6}{100} = 0.06$, $n = 12$, $p = \frac{0.06}{12} = 0.005$, $q = \frac{1}{1+p} = \frac{1}{1.005}$, $F = 800$, $T = 30 \cdot 12 = 360$. So

$$\begin{array}{rcl} P & = & 800\left(\frac{1}{1.005}\right)\left[\frac{\left(\frac{1}{1.005}\right)^{360}-1}{\frac{1}{1.005}-1}\right] \\ & = & \$133,433.29. \end{array}$$