MA 111 Review for Exam 3

Exam 3 (given in class on Thursday, April 15) will cover the consumer price index (adjusting dollar amounts for inflation), interest, annuities (saving money by making monthly investments), loans with repayment plans, and credit cards.

Can you work each homework and worksheet problem *correctly* and *quickly*, providing explanations and justifications, without looking at the text or your notes?

Have you carefully studied the material in the text?

Some practice problems:

1. What does it mean to say that a certain amount of money today may be worth more or less in the past or in the future? Why does this make sense? Answer this question with a few complete sentences.

   The buying power of money changes in the face of inflation. What a dollar could buy in the past is more than what it can buy today. What a dollar can buy today is more than what it will be able to buy in the future.

2. Refer to the Consumer Price Index provided with Worksheet 3.1.

   (a) What is \$2000 in 2010 dollars worth in terms of 1913 dollars?
   \$91.38.

   (b) What is \$2000 in 1913 dollars worth in terms of 2010 dollars?
   \$43,775.15.

   (c) If a certain item cost \$500 in 2000, but costs \$600 today, what is the percent change in the cost once you have adjusted for inflation?
   -4.64%.

3. Simple Interest

   (a) Suppose an amount P is invested at a simple annual interest rate of $R\%$ for t years, yielding a final amount A. Explain how to get the formula $A = P(1 + rt)$, where $r = \frac{R}{100}$.
   Each year an amount equal to rP is added onto the total. If this is done for t years, you then have $P + rPt$, which equals $P(1 + rt)$.

   (b) How much would you have if you invested \$3000 at a simple annual interest rate of 4.5% for 5 years?
   \$3675.

(c) How much must you invest at a simple annual interest rate of 4.5% to end up with \$3000 at the end of 5 years?

\$2448.98.

(d) If you want to invest \$3000 at a simple annual interest rate for 5 years to end up with \$4000, what must the interest rate be?

6.67%.

(e) If you want to invest \$3000 at a simple annual interest rate of 4.5% to end up with \$5000, how many years will you require?

14.81 years.

4. Compound Interest

(a) Suppose an amount P is invested at an annual interest rate of $R\%$ compounded yearly for t years, yielding a final amount A. Explain how to get the formula $A = P(1+r)^t$, where $r = \frac{R}{100}$.
Starting with P, each year you increase your current total by $R\%$ by multiplying the current total by $(1+r)$. If you do this for t years, the net effect is to start with P and multiply it by $(1+r)$ a total of t times yielding the final amount $P(1+r)^t$.

(b) Suppose an amount P is invested at an annual interest rate of $R\%$ compounded monthly for t years, yielding a final amount A. Explain how to get the formula $A = P(1+\frac{r}{12})^{12t}$, where $r = \frac{R}{100}$.
Starting with P, each month you increase your current total by $\frac{R\%}{12}$ by multiplying the current total by $(1+\frac{r}{12})$. If you do this for t years, the net effect is to start with P and multiply it by $(1+\frac{r}{12})$ a total of $12t$ times yielding the final amount $P(1+\frac{r}{12})^{12t}$.

(c) How much would you have if you invested \$3000 at an annual interest rate of 4.5% compounded yearly for 5 years?

\$3738.55.

(d) How much would you have if you invested \$3000 at an annual interest rate of 4.5% compounded monthly for 5 years?

\$3755.39.

(e) How much would you have if you invested \$3000 at an annual interest rate of 4.5% compounded continuously for 5 years? (Formula: $A = Pe^{rt}$.)

\$3756.97.

(f) How much must you invest at an annual interest rate of 4.5% compounded monthly to end up with \$3000 at the end of 5 years?

\$2396.56.

(g) How much must you invest at an annual interest rate of 4.5% compounded continuously to end up with \$3000 at the end of 5 years?

\$2395.55.

(h) Suppose you invest \$3000 for one year at an annual interest rate of 5% compounded monthly. By what percent has your investment grown by the end of the year? This is called the Annual Percent Yield (APY).

5.12%.

(i) Suppose you want to invest \$10,000 at an annual interest rate of 5.25% compounded monthly. How many full years will it take for you to have more than \$20,000?

The "Rule of 70" suggest that it will be about $\frac{70}{5.25} = 13.3$ years. So check both 13 and 14 years in the formula, and you will see that in 13 years your investment will not have yet doubled, but in 14 years you will have \$20821.41. So the answer is 14 years.

5. Annuities

(a) You invest \$200 at the beginning of each month at an annual interest rate of 7% compounded monthly, for a period of 2 years. Explain why the amount of money you will have at the end equals

$$200\left(1+\tfrac{0.07}{12}\right)^{24} + 200\left(1+\tfrac{0.07}{12}\right)^{23} + 200\left(1+\tfrac{0.07}{12}\right)^{22} + \cdots + \\ 200\left(1+\tfrac{0.07}{12}\right)^{3} + 200\left(1+\tfrac{0.07}{12}\right)^{2} + 200\left(1+\tfrac{0.07}{12}\right)^{1}.$$

The first \$200 invested earns interest for 24 months, growing to $200(1+\frac{0.07}{12})^{24}$. The second \$200 invested earns interest for 23 months, growing to $200(1+\frac{0.07}{12})^{23}$. The third \$200 invested earns interest for 22 months, growing to $200(1+\frac{0.07}{12})^{22}$, etc. The last \$200 invested earns interest for 1 months, growing to $200(1+\frac{0.07}{12})^{1}$. Your total in the end is the sum of all of these.

(b) Suppose $S = Aq + Aq^2 + Aq^3 + \cdots + Aq^m$. Show how to get the formula

$$S = Aq\left(\frac{q^m - 1}{q - 1}\right)$$

(assuming $q \neq 1$).

$$S = Aq + Aq^2 + Aq^3 + \cdots + Aq^m.$$

$$Sq = Aq^2 + Aq^3 + Aq^4 + \cdots + Aq^{m+1}.$$

Subtract the first equation from the second to get

$$Sq - S = Aq^{m+1} - Aq.$$

Factor and solve for S to get

$$S(q-1) = Aq(q^m - 1),$$

$$S = Aq\left(\frac{q^m - 1}{q - 1}\right).$$

(c) Suppose you invest \$200 at the beginning of each month at an annual interest rate of 7% compounded monthly for a period of 30 years. How much money will you have in the end?

\$245,417.50.

(d) Suppose you wish to become a millionaire 40 years from now by making a monthly investment of an amount A at the beginning of each month at an annual interest rate of 7% compounded monthly. How much do you need to invest each month?

\$378.77.

(e) Suppose you have the choice to either (a) invest \$100 at the beginning of each month for one year at an annual interest rate of 4% compounded monthly, or (b) invest all of \$1200 right now for one year at an annual interest rate of 3.9% compounded monthly. What is the better choice?

Choice (a) yields \$1226.32. Choice (b) yields \$1247.65. So choice (b) is better.

6. Loans. Formula:

$$L = \frac{A}{q^m}\left(\frac{q^m - 1}{q - 1}\right),$$

where L is the amount of the loan, A is the monthly payment, $q = 1 + \frac{r}{12}$, $r = \frac{R}{100}$, $R\%$ is the annual interest rate, and m is the number of months.

(a) You plan to borrow \$100,000 to set up a 30-year mortgage. The bank can offer you an annual interest rate of 4.5% compounded monthly.

i. What will your monthly payment be?
$506.69.

ii. How much money will you end up paying in interest?
$82,408.40.

(b) You want to borrow some money but realize that you can only afford to make monthly payments of $200 each month.

i. How much money can you borrow if you plan to pay it back over 5 years at an annual interest rate of 3% compounded monthly?
$11,130.47.

ii. How much money will you end up paying in interest?
$869.53.

7. Credit Cards

(a) Assume that you have a credit card with the following terms:

- The APR is 25.75%, but no interest is charged if you pay off your charges in full in the same month that they are made.
- The minimum payment required is the interest accrued that month plus 2.5% of the rest of the balance. However, if this formula yields a minimum payment that is less than $25, the minimum payment will be $25.

Suppose you charge a single item costing $900 and make the required minimum payments. Fill in the following chart for the first two months:

Month	Interest Accrued	Statement	Minimum Payment	Balance
1	$0.00	$900.00	$25.00	$875.00
2	$18.78	$893.78	$41.12	$852.66

Note: I am interpreting the terms of the credit card, as before, as calculating 2.5% of the entire current statement, which includes the accrued interest. It is possible that the terms mean that 2.5% is computed on just the portion of the current statement that does not include the accrued interest. In this case, it takes even longer to pay off the debt!