Worksheet 3.3
Annuities

1. How much money would you have at the end of 12 months if you deposited \$500 at the beginning of each month into an account that earned 5% interest compounded monthly?

Month	Deposit	Number of Months Interest is Earned	Final Amount
1	\$500	12	
2	\$500	11	
3	\$500	10	
4	\$500	9	
5	\$500	8	
6	\$500	7	
7	\$500	6	
8	\$500	5	
9	\$500	4	
10	\$500	3	
11	\$500	2	
12	\$500	1	
Totals		—	

2. In the previous problem, how much did you earn in interest?

3. If you had invested \$6000 all at the beginning of one year at the same interest rate, compounded monthly, how much would you have at the end of the year? Compare this figure to the answer from problem (1). Why is it reasonable that these numbers are different?

4. Solve problem (1) using a spreadsheet. (Please provide me with only the first and last pages of the spreadsheet.)

5. Explain why the solution to problem (1) can be computed as

$$500\left(1+\frac{.05}{12}\right)+500\left(1+\frac{.05}{12}\right)^2+500\left(1+\frac{.05}{12}\right)^3+\cdots+500\left(1+\frac{.05}{12}\right)^{12}.$$

6. If we let $A = 500$ and $q = 1 + \frac{.05}{12}$, then the previous formula becomes

$$Aq + Aq^2 + Aq^3 + \cdots + Aq^{12}.$$

This is an example of a *geometric sum*. Let's call this sum S. Then

$$\begin{aligned} S &= Aq + Aq^2 + Aq^3 + \cdots + Aq^{12}, \\ qS &= Aq^2 + Aq^3 + Aq^4 + \cdots + Aq^{13}. \end{aligned}$$

Show how you can subtract the first equation from the second equation and then simplify a little bit to get the formula:

$$S = Aq\left(\frac{q^{12}-1}{q-1}\right).$$

7. Use this formula to solve problem (1) again. You should get the same answer!

8. Explain where the following formula comes from: If you invest A dollars at the beginning of each month for a total of m months at an interest rate of $R\%$ compounded monthly, then in the end the amount of money you have will be

$$Aq\left(\frac{q^m - 1}{q - 1}\right),$$

where $q = 1 + \frac{r}{12}$ and $r = \frac{R}{100}$.

9. Solve the following problem using (a) the formula and (b) a spreadsheet: How much money would you have at the end of 30 years if you deposited \$500 at the beginning of each month into an account that earned 5% interest compounded monthly? (Please provide me with only the first and last pages of the spreadsheet.)

10. Suppose you want to save up \$50,000 at the end of 20 years by making regular monthly investments of $\$A$ at the beginning of each month at an interest rate of 4% compounded monthly. What must A equal?