Worksheet 3.5
Loans

In an earlier worksheet we looked at annuities — saving money by making regular investments into an account earning interest, and taking the money out at the end. With a small change, we can now analyze loans — receiving money at the beginning, and paying it back with regular payments.

1. Suppose we had a way to invest money at 5% annual interest compounded monthly.

   (a) How much would \$1000 today be worth 12 months from now?

   (b) How much would \$1000 12 months from now be worth today?

2. How much money would you have at the end of 12 months if you deposited \$500 at the *beginning* of each month into an account that earned 5% annual interest compounded monthly? You may use the formula we developed earlier:

$$S = Aq\left(\frac{q^m - 1}{q - 1}\right),$$

   where $q = 1 + \frac{r}{12}$ and $r = \frac{R}{100}$.

3. How much money would you have at the end of 12 months if you deposited \$500 at the *end* of each month into an account that earned 5% annual interest compounded monthly? Note that each deposit now earns interest for one fewer month (the first deposit only earns 11 months interest, and the last deposit earns no interest at all). Explain why you can use the formula:

$$S = A\left(\frac{q^m - 1}{q - 1}\right),$$

where $q = 1 + \frac{r}{12}$ and $r = \frac{R}{100}$.

4. Continuing from the previous problem, how much is the final amount S worth *at the beginning of the first month*? Explain how in general you can just divide S by q^m.

5. Let's call the answer to the previous problem L, for *loan*. This is the amount that you can borrow if you are going to repay the loan according to the given payment plan. So if you borrow an amount L that is to be repaid by paying an amount A at the end of each month for m months at an annual interest rate of $R\%$ compounded monthly, the general formula is

$$L = \frac{A}{q^m}\left(\frac{q^m - 1}{q - 1}\right),$$

where $q = 1 + \frac{r}{12}$ and $r = \frac{R}{100}$.

How much money can you borrow if you plan to repay \$250 at the end of each month for 20 years at an annual interest rate of 4.75% compounded monthly?

6. In the previous problem:

   (a) What is the total amount of your payments?

   (b) How much total interest did you pay?

7. You want to take out a 30 year mortgage on a home by borrowing \$200,000, with payments made at the end of each month at an annual interest rate of 5.25% compounded monthly.

   (a) What is your monthly payment going to be?

   (b) How much total interest are you going to end up paying?

   (c) What other fees are usually associated and combined with your monthly mortgage payment?

8. Using the monthly payment from the previous problem, how much money could you borrow at the same interest rate if you took out a loan for 359 months instead of 360 months? Note: The difference between this number and \$200,000 shows how much of the principal of the original loan you paid in your first month's payment!