Shapes and Designs
Extensions 2

1. Since a triangle is determined (up to congruence) by the lengths a, b, and c of its three sides, so is its area.

   (a) Prove Heron's formula for the area of such a triangle:

   $$A = \sqrt{s(s-a)(s-b)(s-c)},$$

   where s is the *semiperimeter*, $s = \frac{1}{2}(a+b+c)$.

   (b) What happens when you try to apply this formula to an "illegal" triangle—numbers a, b, and c for which the triangle inequality fails?

2. Read the section in *CliffsQuickReview Geometry* on congruent triangles, pp. 43–50. Notice that the SSS, SAS, and ASA congruence criteria are all listed as *Postulates*, which suggests that they cannot be proved from each other. But this is not the case. Assume that the SAS congruence criterion is true as a postulate, and prove the ASA congruence criterion as a theorem.

3. Reconstructing Triangles. Suppose T is a triangle with angles A, B, and C, and respective opposite sides a, b, and c. For convenience we will use the capital letter for both the angle and its measure, and the lower-case letter for both the side and its length.

   (a) SSS Reconstruction. Explain how to use Geometer's Sketchpad to construct a congruent triangle if you are given only a, b, and c.

   (b) SAS Reconstruction. Explain how to use Geometer's Sketchpad to construct a congruent triangle if you are given only a, b, and C.

   (c) SSA Reconstruction. Use Geometer's Sketchpad to explain why we do not have an SSA congruence theorem. Under what additional conditions on the lengths of the given sides is the triangle uniquely determined?

4. Read the section in *CliffsQuickReview Geometry* on Special Quadrilaterals, pp. 60–69.

   (a) Prove Theorems 41–45. You may need to use various results that come earlier in the book.

   (b) Prove Theorems 46–53. You may need to use various results that come earlier in the book.

5. Find one example of a pentagon that tiles the plane (edge-to-edge). Make a careful drawing of the tiling.

6. Use Polydron to construct three-dimensional analogs to the following two-dimensional shapes, and explain clearly why you feel the analogy is reasonable:

   (a) Equilateral triangle

   (b) Isosceles triangle

   (c) Triangle

   (d) Square

   (e) Rectangle

   (f) Rhombus

   (g) Parallelogram

   (h) Trapezoid

   (i) Quadrilateral