Shapes and Designs
Extensions 3

1. Write a clear explanation of the measurement of angles in radians and why this is a more natural notion than measurement in degrees.

2. Propose a definition of the measure of a solid angle where three, four, or more planes meet at common vertex of a polyhedron, and explain why your definition is reasonable. In particular, your definition should be compatible with a three-dimensional analog of the Angle Addition Postulate (*CliffsQuickReview Geometry, p. 12*).

3. Describe all possible cases when two sets (r, θ), (r', θ') of polar coordinates actually correspond to the same point.

4. Review the definitions of trigonometric functions from the unit circle.

   (a) Drawing on this, sketch the graphs of the functions $f(\theta) = \sin\theta$ and $f(\theta) = \cos\theta$, and explain how you can deduce these naturally from the unit circle definition,

   (b) Continuing to think about the unit circle definition, complete the following formulas and give brief explanations for each.

      i. $\sin(-\theta) = -\sin(\theta)$.
      ii. $\cos(-\theta) =$
      iii. $\sin(\pi + \theta) =$
      iv. $\cos(\pi + \theta) =$
      v. $\sin(\pi - \theta) =$
      vi. $\cos(\pi - \theta) =$
      vii. $\sin(\pi/2 + \theta) =$
      viii. $\cos(\pi/2 + \theta) =$
      ix. $\sin(\pi/2 - \theta) =$
      x. $\cos(\pi/2 - \theta) =$
      xi. $\sin^2(\theta) + \cos^2(\theta) =$

5. Describe a procedure to determine the rectangular coordinates (x, y) of a point from its polar coordinates (r, θ) and justify why it works.

6. Cylindrical and Spherical Coordinates

(a) Justify the following conversion from cylindrical coordinates (r, θ, z) to rectangular coordinates (x, y, z).

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{aligned}$$

(b) Justify the following conversion from spherical coordinates (r, θ, ϕ) to rectangular coordinates (x, y, z).

$$\begin{aligned} x &= r\cos\theta\sin\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\phi \end{aligned}$$

7. Read Chapter 2 of *CliffsQuickReview Geometry* on Parallel Lines. Prove Theorems 17–24.

8. It turns out that without assuming Postulates 11 and 12 of *CliffsQuickReview* one can prove that the sum of the measures of the angles of any triangle cannot exceed 180 degrees.

   (a) Learn the proof of this angle sum theorem. See, for example, the proof of the Saccheri-Legendre Theorem in Kay, *College Geometry: A Discovery Approach*.

   (b) Use this result to prove Postulate 12, thereby showing that it was not necessary to assume this as a postulate after all.