Shapes and Designs
Extensions 5

1. Extensions of Problems 5.1 and 5.2 of Investigation 5.

   (a) Generalizing to a regular n-gon, describe all the different ways the n-gon can be placed in its corresponding hole. How many ways are there in all?

   (b) Consider a cube and a cubical hole into which the cube can be inserted. Describe all the different ways the cube can be placed in the hole. How many ways are there in all?

   (c) Repeat the above question for a rectangular solid of dimensions $a \times a \times b$ where $a \neq b$. Of dimensions $a \times b \times c$ where a, b, c are all different.

   (d) Consider a regular tetrahedron (all faces are equilateral triangles). Draw an equilateral triangle of the same size as one of the faces on a sheet of paper. Describe all the different ways the tetrahedron can placed on the sheet of paper to match up with the given triangle. How many ways are there in all?

   (e) Repeat the previous question for an octahedron.

   (f) Repeat the previous question for an icosahedron.

   (g) Repeat the previous question for a dodecahedron (this time drawing a pentagon on the sheet of paper).

   (h) Repeat the previous question for an n-gonal prism.

   (i) How do your answers to all of the above questions change if we also allow the creation of a reflected form of the various polyhedra?

2. Look at ACE Question 1 of Investigation 5.

   (a) How many shortest paths exist from A to C?

   (b) Generalize the previous question if there are m horizontal streets and n vertical streets.

   (c) Generalize the previous question to three dimensions.

3. Look at ACE Question 7 of Investigation 5. Draw some spherical triangles on the surfaces of spheres. What can you say about the angle sums of these triangles?

4. Look at ACE Question 18 of Investigation 5.

   (a) Prove that the path you found to answer this question is indeed the shortest one.

(b) Prove that there is not path from A to C that uses every block side exactly one.

(c) For an arbitrary street network (one in which the streets may not be laid out in a formal horizontal-vertical grid pattern), determine under what conditions it is possible to start at a given intersection point A, find a path that traverses each street between intersections exactly one, and ends at another given intersection point B (which may or may not be the same as A).

(d) Find a description of the problem of the Königsberg bridges.

5. Look at ACE Question 20 of Investigation 5. Solve the following related question:

   (a) How can a rectangle of dimensions $m \times n$ (not necessarily integer or even rational) be cut into a small number of pieces and put back together to make a square? You may assume that the rectangle is not more than twice as long as it is wide. (Why does this end up being a useful assumption?)

   (b) From this, derive the formula for the area of a rectangle from the formula for the area of a square.

6. Look at Mathematical Reflections Question 2. A triangle is *isosceles* if it has at least two sides of equal length. In this case the third side is called the *base* and the two angles of the triangle incident to the base are the *base angles.* Prove Theorems 32 and 34 of *CliffsQuickReview Geometry*, page 50.

7. Learn how to use POV-Ray to draw rectangular solids.