Filling and Wrapping
Extensions

1. Use calculus to derive the formula for the area of a parallelogram of base b and height h.

2. Use calculus to derive the formula for the area of a triangle of base b and height h.

3. Use calculus to derive the formula for the area of a trapezoid with bases b_1, b_2 and height h.

4. Use calculus to derive the area of a circle of radius r.

5. Take the derivative of the formula for the area of a circle of radius r. What formula do you get? Why does this happen?

6. Use calculus to prove Cavalieri's principle: If two 2-dimensional regions have the property that all lines parallel to some fixed line that meet R_1 and R_2 do so in line segments having equal lengths, whose endpoints are the boundary points of R and R', then R and R' have the same area.

7. Two regions R_1, R_2 are *similar* if there is a one-to-one correspondence between the points of R_1 and the points of R_2, and a constant k, such that for every pair of points A, B in R_1 and corresponding pair of points A', B' in R_2, we have $AB = kA'B'$. If R_1 and R_2 are similar 2-dimensional regions, what is the relationship between the areas of R_1 and R_2? (If you are not certain, try this out with simple shapes first.) Use calculus to justify the result.

8. Use calculus to derive the formula for the volume of a pyramid whose base is a polygon of area B and whose height is h.

9. Use calculus to derive the formula for the volume of a cylinder of radius r and height h.

10. Derive the formula of the surface area of a cylinder of radius r and height h.

11. Use calculus to derive the formula for the volume of a cone of radius r and height h.

12. Derive the formula for the surface area of a cone of radius r and height h.

13. Use calculus to derive the formula for the volume of a sphere of radius r.

14. Use calculus to derive the formula for the surface area of a sphere of radius r.

15. Take the derivative of the formula for the volume of a sphere of radius r. What formula do you get? Why does this happen?

16. Use calculus to prove Cavalieri's principle: If two 2-dimensional regions have the property that all planes parallel to some fixed plane that meet R_1 and R_2 do so in plane sections having equal areas, whose boundaries lie in the boundaries of R and R', then R and R' have the same volume.

17. Use Cavalieri's principle to derive the formula for the volume of a hemisphere of radius r by comparing it, "slice by slice," to the volume of a cylinder with radius r and height r from which a cone of radius r and height r has been removed.

18. If R_1 and R_2 are similar 3-dimensional regions, what is the relationship between the volumes of R_1 and R_2? Between the surface areas of R_1 and R_2? Use calculus to justify the result.

19. Suppose a sphere is inscribed in a cylinder whose radius is the radius of the sphere, and whose height is the diameter of the sphere.

    Let R be any region on the sphere. Project this region out horizontally onto the cylinder to get the region R' on the cylinder. Use calculus to prove that R and R' have the same areas. This is one way to make maps of the earth that accurately represent areas of countries, although the shapes of the countries are distorted.

20. Assume that $OABC$ is a tetrahedron with coordinates $O = (0, 0, 0)$, $A = (x_1, y_1, z_1)$, $B = (x_2, y_2, z_2)$, $C = (x_3, y_3, z_3)$. Prove that its volume is

$$\frac{1}{6}\left\| \begin{matrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{matrix} \right\|$$

21. A rectangular box is to be constructed with a surface area of 600 cubic centimeters. What dimensions of the box will minimize its surface area?

22. A soda can is in the shape of a cylinder (with top and bottom) and has a volume of 355 mL. What radius and height should the can have to minimize its surface area?