Looking for Pythagoras
Extensions

1. Suppose you have a rectangular box that with length a, width b, and height c. Find and prove a formula for the length of the interior diagonal connecting two opposite corners.

2.
   (a) Use the Pythagorean Theorem to find and prove the distance formula for the distance between two points (x_1, y_1) and (x_2, y_2) in the plane.

   (b) Repeat the previous problem for two points in three-dimensional space.

3. Consider a tetrahedron with vertices $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$, and $(0, 0, c)$, where $a, b, c > 0$. This seems like a reasonable three-dimensional generalization of a right triangle.

   (a) Consider some specific examples, calculate the areas of the the four faces, and conjecture a generalization of the Pythagorean Theorem.

   (b) Prove this generalization.

4. Three numbers (a, b, c) are called a *Pythagorean triple* if (1) they are all positive integers, and (2) $a^2 + b^2 = c^2$.

   (a) Prove that if $a \geq 3$ is a positive odd integer then $(a, \frac{a^2-1}{2}, \frac{a^2-1}{2}+1)$ is a Pythagorean triple.

   (b) Prove that if $a \geq 4$ is a positive even integer then $(a, \frac{a^2-4}{4}, \frac{a^2-4}{4}+2)$ is a Pythagorean triple.

5. Prove that if ABC is any right triangle with right angle at C, and if the altitude of the triangle from C has length p and divides the hypotenuse of the triangle into two segments of length q and r, then $p^2 = qr$.

6. Look up the arc length formula in calculus and explain what its connection is with the Pythagorean Theorem.

7. Prove that $\sqrt{3}$ is an irrational number.