Connections

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

When students can connect mathematical ideas, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics.

Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. Viewing mathematics as a whole highlights the need for studying and thinking about the connections within the discipline, as reflected both within the curriculum of a particular grade and between grade levels. To emphasize the connections, teachers must know the needs of their students as well as the mathematics that the students studied in the preceding grades and what they will study in the following grades. As the Learning Principle emphasizes, understanding involves making connections. Teachers should build on

students' previous experiences and not repeat what students have already done. This approach requires students to be responsible for what they have learned and for using that knowledge to understand and make sense of new ideas.

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Recognize and use connections among mathematical ideas

By emphasizing mathematical connections, teachers can help students build a disposition to use connections in solving mathematical problems, rather than see mathematics as a set of disconnected, isolated concepts and skills. This disposition can be fostered through the guiding questions that teachers ask, for instance, "How is our work today with similar triangles related to the discussion we had last week about scale drawings?" Students need to be made explicitly aware of the mathematical connections.

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The notion that mathematical ideas are connected should permeate the school mathematics experience at all levels. The mathematical experiences of children first entering school have not been separated into categories, and this integration of mathematics in many contexts should continue in school. Children can learn to recognize mathematical patterns in the rhythms of the songs they sing, identify the hexagonal shape in a honeycomb, and count the number of times they can jump rope successfully. As students move into grades 3–5, their mathematical activity should expand into more-abstract contexts. They can begin to see the connections among arithmetic operations, understanding, for example, how multiplication can be thought of as repeated addition. As they see how mathematical operations can be used in different contexts, they can develop an appreciation for the abstraction of mathematics. In grades 6–8, students should see mathematics as a discipline of connected ideas. The key mathematical ideas in the middle grades are themselves closely connected, and ideas about rational numbers, proportionality, and linear relationships will pervade much of their mathematical and everyday » activity. In grades 9–12, students not only learn to

expect connections but they learn to take advantage of them, using insights gained in one context to solve problems in another.

Throughout the pre-K–12 span, students should routinely ask themselves, "How is this problem or mathematical topic like things I have studied before?" From the perspective of connections, new ideas are seen as extensions of previously learned mathematics. Students learn to use what they already know to address new situations. Elementary school students link their knowledge of the subtraction of whole numbers to the subtraction of decimals or fractions. Middle-grades students recognize and connect multiple representations of the same mathematical idea, such as the ratio that represents rate of change and the tilt or slope of a line. High school students connect ideas in algebra and geometry.

Some activities can be especially productive for featuring mathematical connections. For instance, the relationship between the diameter and the circumference of a circle can be studied empirically by collecting a variety of circular objects and measuring their circumferences and diameters. Middle-grades students might collect and graph data for the two variables—circumference (C) and diameter (d). By doing so, they can see that all the points lie close to a straight line through (0, 0), which suggests that the ratio of C/d is constant. This activity usually leads to an average value for C/d that lies between 3.1 and 3.2—a rough approximation of π. The problem involves ideas from measurement, data analysis, geometry, algebra, and number.

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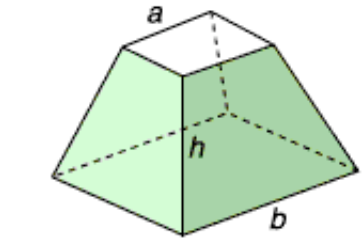
Understand how mathematical ideas interconnect and build on one another to produce a coherent whole

As students progress through their school mathematics experience, their ability to see the same mathematical structure in seemingly different settings should increase. Prekindergarten through grade 2 students recognize instances of counting, number, and shape; upper elementary school students look for instances of

arithmetic operations, and middle-grades students look for examples of rational numbers, proportionality, and linear relationships. High school students are ready to look for connections among the many mathematical ideas they are encountering. For instance, a method for finding the volume of the truncated square pyramid shown at the top of figure 3.7 is suggested by the method for finding the area of the trapezoid that is also shown in the figure (Banchoff 1990, pp. 20–22).

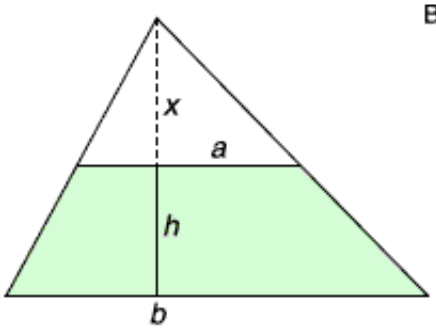


$$\text{Area of small triangle} = \frac{1}{2}ax.$$

$$\text{Area of large triangle} = \frac{1}{2}b(x+h).$$

$$\text{Area of trapezoid} = \frac{1}{2}b(x+h) - \frac{1}{2}ax$$

$$= \frac{1}{2}bx + \frac{1}{2}bh - \frac{1}{2}ax.$$



By similarity of triangles,

$$\frac{x}{(x+h)} = \frac{a}{b}$$

$$bx = a(x+h)$$

$$\text{Area of trapezoid} = a(x+h) + \frac{1}{2}bh - \frac{1}{2}ax$$

$$= \frac{1}{2}h(a+b).$$

Fig. **3.7.** Connections between methods for finding the volume of a truncated pyramid and for finding the area of a trapezoid

As students develop a view of mathematics as a connected and integrated whole, they will have less of a tendency to view mathematical skills and concepts separately. If conceptual understandings are linked to procedures, students will not perceive mathematics as an arbitrary set of rules. This integration of procedures and concepts should be central in school mathematics.

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Recognize and apply mathematics in contexts outside of mathematics

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School mathematics experiences at all levels should

include opportunities to learn about mathematics by working on problems arising in » contexts outside of mathematics. These connections can be to other subject areas and disciplines as well as to students' daily lives. Prekindergarten through grade 2 students can learn about mathematics primarily through connections with the real world. Students in grades 3–5 should learn to apply important mathematical ideas in other subject areas. This set of ideas expands in grades 6–8, and in grades 9–12 students should be confidently using mathematics to explain complex applications in the outside world.

The opportunity for students to experience mathematics in a context is important. Mathematics is used in science, the social sciences, medicine, and commerce. The link between mathematics and science is not only through content but also through process. The processes and content of science can inspire an approach to solving problems that applies to the study of mathematics. In the *National Science Education Standards,* a yearlong elementary school science activity about weather is described (National Research Council 1996, pp. 131–33). The connections to mathematics in this activity are substantial: students design instruments for measuring weather conditions and plan for how to organize and communicate their data.

Steinberg (1998, p. 97) reports the following incident in which eleventh-grade students at a high school worked with the CVS Corporation to locate a new pharmacy in a Boston neighborhood:

> Although fully aware that the company would probably not rely only on their calculations in making a monetary decision as to where to locate a store, the students still felt involved in a real problem.... Organized into small work teams supported by experts from various departments within the CVS organization, students analyzed demographic and economic data to determine market demand for a CVS pharmacy in different neighborhoods. Students also worked with CVS staff to identify and evaluate several possible locations for the new store.... Students worked with architects on design options for the new store and worked with accountants on financing plans.

This project was incorporated into the students' mathematics and humanities classes. The students saw the connections of mathematics to the world of commerce and to other disciplines, and they also saw the connections within mathematics as they applied knowledge from several different areas.

Data analysis and statistics are useful in helping students clarify issues related to their personal lives. Students in prekindergarten through grade 2 who are working on calendar activities can collect data on the weather by recording rainy, cloudy, or sunny days. They can record the data, count days, generalize about conditions, and make predictions for the future. Students in grades 3–5 can use the Internet to collaborate with students in other classrooms to collect and analyze data about acid rain, deforestation, and other phenomena. By grades 9–12, students should be able to use their knowledge of data analysis and mathematical modeling to understand societal issues and workplace problems in reasonable depth.

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