MA 310 — Homework #3
Solution

Solve this variant of the census taker problem.

How Many Children? "I hear some youngsters playing in the back yard," said Jones, a graduate student in mathematics. "Are they all yours?" "Heavens, no," exclaimed Professor Smith, the eminent number theorist. "My children are playing with friends from three other families in the neighborhood, although our family happens to be largest. The Browns have a smaller number of children, the Greens have a still smaller number, and the Blacks the smallest of all." "How many children are there altogether?" asked Jones. "Let me put it this way," said Smith. "There are fewer than 18 children, and the product of the numbers in the four families happens to be my house number which you saw when you arrived." Jones took a notebook and pencil from his pocket and started scribbling. A moment later he looked up and said, "I need more information. Is there more than one child in the Black family?" As soon as Smith replied, Jones smiled and correctly stated the number of children in each family. Knowing the house number and whether the Blacks had more than one child, Jones found the problem trivial. it is a remarkable fact, however, that the number of children in each family can be determined solely on the basis of the information given above. What are these numbers?

Solution. Since Jones needed further information, we know that he found several combinations of four different numbers summing to less than 18 that had the same product, and for which the smallest numbers were not all the same. By exhaustively checking all combinations for which these conditions are met (which must be done carefully), one is led to the three combinations with product 120; namely, (1,3,5,8), (1,4,5,6), and (2,3,4,5). Further, since Jones could determine the correct combination by knowing whether the smallest number was 1 or not, we know that the answer must not have been 1; otherwise Jones would have to ask yet another question. So the correct combination is (2,3,4,5).