

Positions and Their Types

Cut-Cake

“In this normal-play game played between Louise and Richard, each position consists of a collection of uncut pieces of cake. Each uncut piece of cake is rectangular and has dotted lines running horizontally and vertically indicating where it can be cut. On her turn, Louise chooses any piece of cake and makes a vertical cut, while on Richard’s turn he chooses any piece and makes a horizontal cut. The last player to make a move wins.”

Domineering

“This is a normal-play game played using some squares from a rectangular array. On Louise’s turn, she may place a 2×1 [vertical] domino over two unoccupied squares. On Richard’s turn, he may place a 1×2 [horizontal] domino over two unoccupied squares. Since this is a normal-play game, the last player to move wins.”

LRNP

Consider games with no ties in which the last player to move wins (normal play). Such games each have types:

- Type L. L has a winning strategy whoever goes first.
- Type R. R has a winning strategy whoever goes first.
- Type N. The Next player to move has a winning strategy.
- Type P. The second (or Previous) player to move has a winning strategy.

Questions

1. What is the type of the position of Pick-Up-Bricks consisting of five bricks?
2. What is the type of the position of Pick-Up-Bricks consisting of six bricks?
3. Find a Domineering position of type L with at most four squares. Repeat for types R, N, and P.
4. Find a Cut-Cake position of type L with at most four squares. Repeat for types R, N, and P.
5. Consider a starting position in a normal play game.
 - (a) Suppose that L has a position of type L or P that she can move to, and R has a position of type R or P that he can move to. Explain why the position of the game is of type N.
 - (b) Suppose that L has a position of type L or P that she can move to, but R can only move to positions of type L or N. Explain why the position of the game is of type L.
 - (c) Suppose that L can only move to positions of type R or N, but R has a position of type R or P that he can move to. Explain why the position of the game is of type R.
 - (d) Suppose that L can only move to positions of type R or N, and R can only move to positions of type L or N. Explain why the position of the game is of type P.
6. Use your solution to the previous problem to find the types of the positions below.
 - (a) A 2×4 position in Cut-Cake.
 - (b) A blank 2×3 board in Domineering.