MA 327/ECO 327
Exam #1 Tips

Review the material in Chapters 1 through 4, the class notes that are on the course website, the homework problems and their solutions that are in Canvas, and the various handouts that are on the course website.

Chapter 1. Be able to:

1. Describe the characteristics of combinatorial games.
2. Analyze and directly describe winning strategies for simple specific games such as the Game of 100 and Tic, and others.
3. Represent a game as a W-L-D game tree.
4. Describe what a strategy on a W-L-D tree is, and what it means for a strategy to be winning for L, winning for R, drawing for L, or drawing for R.
5. Completely label a W-L-D game tree by working backwards, labeling each node with $+-$, $-+$, or 00, as well with an appropriate choice of move.
6. Know Zermelo's Theorem and that this is a consequence of the above labeling. Every W-L-D game tree is one of three types.
    - $+-$. L has a winning strategy; i.e., by following this strategy, L will win no matter what R does.
    - $-+$. R has a winning strategy; i.e., by following this strategy, R will win no matter what L does.
    - 00. Both players have drawing strategies; i.e., by following this strategy, L will not lose no matter what R does, and R will not lose no matter what L does,
7. Describe types of strategies such as symmetry, pairing, and strategy stealing.
8. Describe strategies (choices of moves) of certain games without drawing the entire game tree, including Pick-up-Bricks, Chop, and the Game of 100.
9. Justify that for every rectangular position in Chomp except one by one, the first player has a winning strategy.

10. Justify that the first player has a winning strategy in Hex.

Chapter 2. Be able to:

1. Define normal play combinatorial games.
2. Define types L, R, N, and P.
3. Analyze the types of specific positions in certain games such as Cut-Cake and Domineering.
4. Represent normal play games as an ordered pair of sets of positions for L and for R.
5. Determine the types of positions by working your way up from simpler positions to more complicated positions.
6. Describe sums of games.
7. Prove that if β is type P then α and $\alpha + \beta$ are the same type.
8. Prove that if α and β are both type L, then $\alpha + \beta$ is type L. Similarly, if α and β are both type R, then $\alpha + \beta$ has type R.
9. Analyze cases of positions in which the types of other sums are ambiguous.
10. Define what it means for two games to be equivalent.
11. Verify for certain games whether they are or are not equivalent.
12. Understand that equivalence of games is an equivalence relation. If α, β, and γ are positions in normal-play games, then
    (a) $\alpha \equiv \alpha$ (Reflexive Property).
    (b) $\alpha \equiv \beta$ implies $\beta \equiv \alpha$ (Symmetric Property).
    (c) $\alpha \equiv \beta$ and $\beta \equiv \gamma$ implies $\alpha \equiv \gamma$ (Transitive Property).
13. Give an example of two games that that have the same type but are not equivalent.
14. Prove that: If β is of type P, then $\alpha + \beta \equiv \alpha$.
15. Prove that: If α and α' are type P, then $\alpha \equiv \alpha'$.

Chapter 3. Be able to:

1. Define what it means for a game to be impartial.
2. Explain why all positions of impartial games are of type P or N, and the winning strategy is to always move to a P position.
3. Describe the game $*a$.
4. Know that every impartial game is equivalent to some $*k$, where k is a nonnegative integer called the nim or Grundy value of the game, and that a position has type P if and only if the position has value zero.
5. Define and use the MEX rule to determine the nim value of a position.
6. Compute nim sums and know that the nim value of the sum of two games is the nim sum of their values.
7. Use a binary balancing strategy to identify P and N positions in Nim and in other impartial games, including sums of games, and to move to balanced positions.
8. Analyze specific positions using these methods.
9. Know the nim values of positions in Multi-Pile Nim, Chop, and Pick-up-Bricks.

Chapter 4. Be able to:

1. Define what it means for a game to be partizan.
2. Know that certain games behave like numbers with respect to addition and can be assigned values.
3. Compute the values of specific positions by adding particular games and using direct analysis of strategy.
4. Define dyadic numbers and find the simplest dyadic number in an interval.
5. Determine the value of the game by using the simplicity rule, working your way up from less complicated positions to more complicated positions.

6. Determine the best move to make based on its value and the values of its subsequent positions.
7. Compute the value of a Checkers Stacks position using the short cut rule.
8. Create a position in Checker-Stacks or multi-pile Checker-Stacks for any given dyadic number.