MA341 Homework #9

The final project is due on the last day of class. It should be something simple but pretty made with POV-Ray (preferably), Maple, or a physical model. A simple "still-life" consisting of some polyhedra would be fine, but use a polyhedron that we did not already create with the computer. Try some octahedra, pyramids, or perhaps some cuboctahedra. Email me the POV-Ray or Maple source file so that I can create the object as well. You may work in pairs. On Wednesday, November 28, give me a brief written statement of what you intend to create.

I will collect notebooks on Monday, December 3 and return them on Wednesday, December 5. Read the instructions in the course syllabus for the format. Be sure you have a detailed index, and not simply listing "Homework #1," "Homework #2," etc. If you are not certain whether your index is detailed enough, email it to me in advance for my comments.

Five types of transformations of a figure:

Identity: Each point of the figure remains fixed.

Translation: Each point of the figure is moved in the same direction by the same amount. The translation is *trivial* if the points are not moved at all (i.e., if the translation is actually the identity); otherwise, it is *nontrivial*.

Rotation: Each point of the figure is rotated about the same center of rotation by the same angle. The rotation is *trivial* if the rotation angle is a multiple of 360 degrees (i.e., if the rotation is actually the identity); otherwise, it is *nontrivial*.

Reflection: Each point of the figure is reflected across the same given line.

Glide Reflection: Each point of the figure is reflected across the same given line, and then translated by the same amount *in a direction parallel to the reflection line.* The glide reflection is *trivial* if the translation amount is zero (i.e., if the glide reflection is actually a reflection); otherwise, it is *nontrivial*.

1. Carry out the indicated transformations on the attached pages.

2. On the attached pages are several examples of transformations. In each case, identify the type of transformation used to transform figure A into figure B. If the transformation is a translation, determine the direction and amount of translation and indicate it by drawing an arrow. If the transformation is a rotation, determine the center of rotation and the angle of rotation. Mark the center of rotation and draw the angle of rotation at that point. If the transformation is a reflection, determine the line of

reflection and draw it. If the transformation is a glide reflection, determine the line of reflection and the amount of translation, draw the line of reflection, and indicate the translation by an arrow. *In each case explain in words how you determined the various lines, points, angles, and translations—it should not be by guessing. You will need to use a compass and straight-edge.*

3. What are all the possible outcomes when one transformation is followed by another? Fill in the following chart with the letters (a) through (e), corresponding to the following items. There is often more than one answer. The first transformation is indicated by the row and the following transformation is indicated by the column.

   (a) Identity
   (b) Nontrivial Translation
   (c) Nontrivial Rotation
   (d) Reflection
   (e) Nontrivial Glide Reflection

	Translation	Rotation	Reflection	Glide Reflection
Translation				
Rotation				
Reflection				
Glide Reflection				

   Justify each of your conclusions of the second row (rotation followed by another transformation) with a small *careful* sketch.

4. Carefully justify why a reflection in one line ℓ_1, followed by a reflection in another intersecting line ℓ_2, is a rotation about the point of intersection by an angle double that of the angle from ℓ_1 to ℓ_2.

5. Carefully justify why a reflection in one line ℓ_1, followed by a reflection in a parallel line ℓ_2, is a translation by an amount equal to twice the distance between the two lines.

6. Complete Problem 4.1 (Labsheet 4.1B) and Problem 4.2 (Labsheet 4.2B) on pages 60 and 61 of *Kaleidoscopes, Hubcaps, and Mirrors*.

7. Complete the Great Pattern Classification Challenge. Use the flowchart to classify the 17 given patterns according to their symmetries—there is exactly one of each type. You may assume that each pattern extends forever in all directions. Note: in the flowchart, “axis” means “line.”