

MA 341 — Log of Class Activities

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1 Wednesday, August 22

1. Watched the trailer for the movie *Contact*. Discussed Problems 1.1.1 and 1.1.2. The last part of the message in Problem 1.1.1 is the formula for the volume of a particular sphere, and it was suggested to use the radius of the sun as the length of reference. Handout: Pages 1–2 of the class notes.
2. Went over the syllabus (handout).
3. Mentioned a few places where the number e arises “naturally.”
 - (a) If the derivative of the function $f(x) = a^x$ is itself (and $a \neq 0$), then $a = e$.
 - (b) If you invest \$1 into an account yielding the incredible growth rate of 100% per year, if the interest is compounded continuously, you would end up with \$ e at the end of one year.
 - (c) e arises in probability theory too. (I did not mention this in class, but for example, if you have n letters associated with n envelopes, and the letters are inserted at random, the probability of no letter ending up in the correct envelope approaches $1/e$ as n approaches infinity.)
 - (d) The first homework assignment will be emailed out today and posted to the courses website.

2 Friday, August 24

1. Showed the video *Pipedreams* produced by Animusic. This is a nice example of applied geometry, using digital modeling for geometric objects, and applying transformations. We will hopefully have some encounters in our course with creating digital images.
2. Discussed the following problem: Given:

$$a = x^2$$

and

$$b = y^2$$

conclude

$$a - b = (x - y) * (x + y).$$

One important observation was that there are many choices of interpretation of the meanings of the letters—integers, real numbers, complex numbers, matrices, functions, and more. If the assignment of meaning to the letters is compatible with the assumptions, then the conclusion automatically follows, without having to prove it over and over for each context.

3. Discussed Problems 1.1.3 and 1.1.4. We came up with several different representations for toves, borogoves, and gimbling
 - (a) A diagram with points for toves and borogoves, connecting a tove with a borogove if they gimble.
 - (b) A diagram with points for toves, connected two toves if there is a borogove that gimbles with those two toves.
 - (c) A chart, or matrix, with rows labeled by toves and columns by borogoves. If a tove t_i gimbles with a borogove b_j , then we place a 1 in that row and column. The resulting matrix will have exactly two 1's in each column.

We also thought about endowing the nonsense terms with the meaning that toves are people and borogoves are handshakes. Thinking this way we were able to argue that if you add up the number of handshakes h_i that each person i makes, getting $h_1 + \dots + h_n$, then this must equal twice the number of handshakes. As a result, the number of odd numbers in the list h_1, \dots, h_n must be even. This argument can be rephrased to prove the desired result in the abstract context of the undefined terms of toves, borogoves, and gimbling. (P.S. Does everyone know where these nonsense terms come from?)

4. We concluded with a brief discussion of how we can use the result of Problem 1.1.3 to address Problems 1.1.5, 1.1.6, and 1.1.7.
5. I asked that class members attempt to install SketchUp on their laptops and bring them to class on Monday.

3 Monday, August 27

1. Demonstrated the program SketchUp, which has a free version that you can download. There are links on the course website, including links to an introduction and to the Jessie Clark Middle School project with the eighth graders that I worked with a few years ago. You can export images from SketchUp to paste into other types of documents, such as Word. At the SketchUp website there are also links to video tutorials, and to work of students.
2. Discussed features of axiomatic systems. Handout: pages 4–6 of the course notes. Discussed part of Problem 1.2.1. In passing mentioned also that the mathematician Gödel proved that any axiomatic system that includes the arithmetic of integers is necessarily incomplete, and that its consistency is unprovable within the system. Next time we will start with Problems 1.1.2–1.1.5.

4 Wednesday, August 29

1. Collected Homework #1 and passed out the assignment for Homework #2 (also posted on the course website and linked to Blackboard).
2. Worked on Problems 1.1.2–1.1.5, which are part of Homework #2.

5 Friday, August 31

1. Played a geometry video, made from hundreds of images found by a search for “geometry” in Google images. This is partly for fun, but also partly to see the tremendous range of the presence of geometry in human activity.
2. Gave an introduction to GeoGebra, available for free download (and you should get it!). There are lots of tutorial materials on the internet. Demonstrations included constructing the circumcenter and circumscribed circle of a triangle using the common intersection point of the three side bisectors, constructing the incenter and inscribed circle of a triangle using the common intersection point of the angle bisectors, and constructing the tangent line to a parabola. I mentioned that it is also possible to visualize eigenvectors, and I have posted a GeoGebra file and an applet exported from this file onto the course website (under GeoGebra).
3. Mentioned the program Wingeom, which only runs in Windows, but does have a 3D window, as well as a spherical window. There is a link to this program on the course website. GeoGebra is working on a beta test of a 3D window.
4. Mentioned that the Pythagorean Theorem can be interpreted as involving actual square polygons attached to the three sides of a right triangle, and considered a visual proof.
5. Had some general discussion about the use of technology to assist learning in math courses. If anyone has some specific things they would like to discuss further (e.g., math topics that they would like to see visualized), just let me know.

6 Wednesday, September 5

1. Worked on Problem 1.3.1 from the Course Notes. Handout: Pages 9–10.
2. Returned Homework #1, collected Homework #2, and assigned Homework #3.
3. Briefly discussed finite projective geometries. Showed the iPad app for the game of SET (which can also be purchased as a deck of cards). This can be viewed as an example of a finite geometry with 81 points, in which every pair of points determines a line (a particular set of three cards.) See links on course website.
4. Gave a demonstration of using GeoGebra to visualize the result of multiplying a vector by a 2×2 matrix, including eigenvalues and eigenvectors. There is a link to this on the course website.
5. Began discussion of Milestones in Geometry, from Course Notes. Handout: Pages 14–15.

7 Friday, September 7

1. Mentioned that SketchUp should already be installed in the Architecture and in the King Science computer labs. I have requested that it also be installed in the Young Library labs, along with GeoGebra being installed there and in the King Science lab.
2. Finished up discussion of Milestones in Geometry. The Wikipedia article on “History of Geometry” is pretty good, if you want to read more.
3. Emphasized that in our course we will try to integrate aspects of synthetic geometry (like Euclid and the initial SMSG geometry books) with aspects of analytic geometry (geometry with coordinates), calculus (a powerful tool for areas, volumes, and surface areas), and matrix algebra (for transformations).
4. Discussed some examples from the *Common Core State Standards for Mathematics* that some might not be comfortable or familiar with (part of Homework #2). Some examples:
 - (a) The origins of sine and cosine and the like. Mentioned that if you draw a circle of radius 1 centered at the origin, and draw a radius, making an angle α with respect to the positive x -axis, then the intercepted point on the circle has coordinates $(\cos \alpha, \sin \alpha)$. We will spend some time on trigonometry.
 - (b) Arcs and angles in circles. Mentioned that you take a string whose length is the radius of a circle, and then wrap it around a part of that circle, the central angle that it corresponds to (subtends) is 1 radian, by definition. So, because the circle has circumference $2\pi r$, there are going to be a total of 2π radians around the circle. we will return to this in the course.
 - (c) Constructing proofs. We will not go through all of the proofs of Euclidean geometry, but will be doing lots of proving and justification in this course.
 - (d) Congruence. We will discuss this in the course, including connections with transformations.
5. Used Jing to make a video on how to construct an equilateral triangle according Euclid, Book I, Proposition 1—refer to the link on the course website for Euclid’s *Elements*. I used GeoGebra for this, and also explained how to create a new GeoGebra tool for making equilateral triangles. I will modify this video and then post it to the course website. I will also post a link to Jing, which can be used to make videos of your computer screen. Demonstrated how to construct an angle bisector and a

perpendicular bisector of a line segment—see Euclid, Book I, Propositions 9 and 10 and the discussion on the *Elements* website.

6. Mentioned that Dropbox is a very useful free program to share files with emailing. I will put a link to Dropbox on the course website.

7. Worked on the following problems.

(a) Algebraically describe the line passing through the points $(1, 3)$ and $(3, -8)$. Some solutions:

i. The set of points satisfying the equation $y - 3 = \frac{-11}{2}(x - 1)$. This equation is in *point-slope form*,

ii. The set of points satisfying the equation $y = \frac{-11}{2}x + \frac{17}{2}$. This is the *slope-intercept form*.

iii. The set of points satisfying the equation $\frac{11}{2}x + y = \frac{17}{2}$ or the equation $11x + 2y = 17$. These equations are in *standard form*.

(b) Algebraically describe the line passing through the points $(1, 3)$ and $(3, 3)$.

One solution: The set of points (x, y) satisfying the equation $y = 3$. When graphed, this is a horizontal line.

(c) Algebraically describe the line passing through the points $(1, 3)$ and $(1, -8)$.

One solution: The set of points (x, y) satisfying the equation $x = 1$. When graphed, this is a vertical line.

8 Monday, September 10

1. Confirmed that the first Exam will be on Wednesday, September 19.
2. Made a correction of on Homework #3, Problem 3c(ii): “Which is the first Proposition that makes use of the Parallel Postulate in its proof? (Look at the right-hand margins of the Propositions in Book I.)”
3. Discussed pages 23–30 in the course notes. In particular, discussed Problems 2.1.3, 2.1.4. We did 2.1.5 last time, and we also mentioned the point-slope and slope-intercept forms of lines last time. We discussed Theorem 2.1.8 and Problems 2.1.9 and 2.1.10. Then we jumped to Sections 2.2 (but did not define 4×4 determinants).

9 Wednesday, September 12

1. Mentioned the series “Engines of our Ingenuity” as a very nice resource. I placed a link to the episode on Descartes on the course website.
2. Showed the video by Vi Hart on the Origami Proof of the Pythagorean Theorem, with link on the course website.
3. Mentioned my Math Education and Outreach website, with link on the course website.
4. Discussed what it means for an axiomatic system for people and committees to be categoric, with examples from homework. To be categoric means that for every pair of two models whatsoever, it is possible to rename the people in one model so the you exactly match the list of committees in the other model.
5. Passed out review sheet for Exam #1 — see link on course website. Exam #1 will be on Wednesday, September 19.
6. Returned Homework #2 and collected Homework #3.
7. Discussed how to calculate 4×4 determinants.
8. Discussed Theorem 2.4.1 — used for testing collinearity of three points.

10 Friday, September 14

1. Solutions to Homework #3 will be posted on the course website.
2. Worked an example of the collinearity test, Theorem 2.4.1 in the Course Notes posted on the website.
3. For those familiar with matrix algebra (MA 322), mentioned the geometric interpretation of the collinearity test in terms of linear dependence of three vectors in \mathbf{R}^3 .
4. Reviewed matrix multiplication and how to express a system of two linear equations in matrix form. Discussed the inverse of a 2×2 matrix.
5. Discussed, used, and proved Cramer's Rule for the intersection of two lines, Theorem 2.5.1 in the Course Notes.

11 Monday, September 17

1. Returned Homework #3.
2. Answered questions relating to the upcoming exam on Wednesday.
3. Discussed the parametric form of a line, which can be determined from two points. See Section 2.6 of the Course Notes. We did not prove Theorem 2.6.1. We discussed how different values of t correspond to some specific points; e.g., $t = \frac{1}{2}$ corresponds to the midpoint of the segment joining the two given points. If you divide the direction vector (u, v) by its length, you get a direction vector of length 1. Then the values of t correspond to a faithful coordinate system attached to the line.
4. Demonstrated how to illustrate the parametric form of a line with GeoGebra. You can make a slider for the number t , plot two points A and B , and then type $A + t(B - A)$ into the input line to plot the general point on the line, C . You can turn the trace on for C by “right click” or “control click”, and then as you move the slider the point C will trace out a line. I will post the sketch on the course website.

12 Wednesday, September 19

1. Exam #1.

13 Friday, September 21

1. Returned Exam #1, assigned Homework #4.
2. Introduced some resources related to the concept of distance, including
 - (a) A Question of Scale — handout from the Lawrence Hall of Science.
 - (b) Scale of the Universe — website.
 - (c) Powers of Ten — video.
 - (d) Cosmic Voyage — more recent video in the spirit of Powers of Ten.
 - (e) Theodolite and EasyMeasure — iPad apps for measuring distance.

I put links on the course website for these, including a list of some other iPad apps useful in math and science, including WolframAlpha, 123D Catch, and Geometry Pad.

14 Monday, September 24

1. Began with a discussion concerning the set $\{(x, y) : (x - 1)^2 + (y - 2)^2 = 63\}$. This led to many interconnecting ideas, such as recognizing that this is the equation of a circle with center $(1, 2)$ and radius $\sqrt{63}$, this equation comes from a direct application of the distance formula, and the distance formula comes from a direct application of the Pythagorean Theorem. We also noted how to find the x - and y -intercepts, that when solving for y we have two equations due to the taking of a square root—that circle itself is the union of the graph of two functions, and that the original equation itself can be entered into and graphed with such software as GeoGebra and WolframAlpha.
2. Worked on problems 1–3 of Homework #4.
3. Mentioned the role of formative assessment. I put a link to the NCTM Research Briefs on the course website for more information, but to summarize, formative assessment includes assessment during the teaching and learning process, providing opportunities for the teacher to make adjustments as may be necessary. Summative assessment is carried out at the end of the process.
4. I asked everyone to look at problem 4 of Homework #4 before next class.

15 Wednesday, September 26

1. Demonstrated the Socrative software—a “clicker” system that can be used with up to 50 participants.
2. Mentioned the iPad app Splashtop to control a computer from your iPad.
3. Worked on problem 4 of Homework #4, including how an algebraic formulation using the distance formula and calculus leads to a consideration of similar triangles and a shortcut to the solution.

16 Friday, September 28

1. Solved the following problem: Given points $A = (-3, 0)$ and $B = (3, 0)$, find a simple equation to describe the set of point $P = (x, y)$ such that $|PA - PB| = 2$. We set this up using the distance formula, squared both sides, isolated the resulting square root on one side, squared both sides again, and simplified (a lot!) to arrive at a very simple equation, which describes a hyperbola.
2. Demonstrated how to visualize conic sections—actual cross sections of a double cone with a plane—using SketchUp. First the cones are made by revolving triangles around the z -axis, using the “Follow Me” tool. Then a section plane is created and maneuvered to slice the double cone in various ways, to create parabolas, hyperbolas, circles, and ellipses. It is also possible to get a pair of crossing lines, as well as a single point.
3. Began to discuss measuring lengths of curves. There are challenges when measuring real objects with fractal nature, like coastlines. See Section 3.7 in the course notes.

17 Monday, October 1

1. Exam #2 will NOT be on October 10. I need to reschedule this.
2. The assignment for Homework #5 will be sent out shortly.
3. Collected Homework #4. I will write up and distribute solutions.
4. Passed out 11 pages of notes on angles and trigonometry—pages 65–75 of the Course Notes, posted to the course website.
5. Showed the SketchUp file for conic sections, posted to the course website.
6. Discussed the “camper” problem a little more (from Homework #4), and showed how the problem can be completely solved by reflecting the point B across the line $y = 0$ and joining A to this new point. This also relates to the fact that the angle of incidence equals the angle of reflection when light is reflected by a mirror. The calculus approach is still valuable, however, and also can be used to prove Snell’s law for refraction.
7. Showed the Snowflake Curve, which is a fractal of infinite length that bounds a finite area. Refer to the Wikipedia site for more details and illustrations.
8. Demonstrated with GeoGebra the notion that the tangent line to a curve becomes an excellent approximation for small changes in x —just zoom in enough and the curve becomes virtually indistinguishable from the tangent line. This is a good image when thinking about begin differentiable, using the tangent line to approximate the function, and using differentials to approximate nearby values of a function.
9. Discussed how to use the distance formula to motivate the arclength formula of calculus,

$$\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

Mentioned that the St. Louis arch is based on a catenary, for which this formula is not difficult to use. The general equation of a catenary is

$$y = \frac{a}{2}(e^{x/a} + e^{-x/a}).$$

10. Showed the video on Infinity Elephants; the link is posted on the course website.
11. Defined the notion of radian, and the radian measure of an angle. This also motivates the conversions between radians and degrees.

18 Wednesday, October 3

1. Discussed the origin of the names of the trigonometric functions—see the links that I have added to the course website.
2. Defined sine and cosine using the coordinates of points on a unit circle centered at the origin. Illustrated this with an animated GeoGebra sketch, posted to the course website.
3. Worked on Problem 6.3.3 from the course notes. In your sketches, please make your angle α be closer to 20 or 30 degrees rather than 45 degrees. It is also perhaps wiser to label the coordinates of the point on the circle (a, b) instead of the generic (x, y) .

19 Friday, October 5

1. Introduced the NCTM Illuminations website, with lots of activities and lessons, including one on the distance to the horizon. I will post links on the course website.
2. Gave some examples of how knowing coordinates can help you make some striking images with the use of the free ray-tracing software POV-Ray. I will post links on the course website.
3. Discussed the polar coordinates of a point $P = (x, y)$: If the ray from the origin O to the point P makes the angle α with respect to the x -axis, and the length OP equals r , then the polar coordinates of P are (r, α) . We saw why it made sense that $x = r \cos \alpha$ and $y = r \sin \alpha$.
4. Mentioned how to derive the sine and cosine of common angles like $\pi/6$, $\pi/4$, and $\pi/3$.
5. Worked on Problem 6.3.3. Again, you can make sense of everything by thinking about the unit circle.

20 Monday, October 8

1. Passed out and briefly discussed the “Theodolite” problem, also posted to the course website. This is a nice application of trigonometry to determine height and distance of an object, as the iPad Theodolite app does.
2. Gave some pointers on doing Problem 6.4.7. But this should be straightforward.
3. Discussed Problem 6.4.8 in some detail.

21 Wednesday, October 10

1. Returned Homework #4. Solutions are posted on the course website.
2. Collected Homework #5. Solutions are now written into the Course Notes.
3. Reminder of Exam #2 on October 17. Exam review sheet is posted on the course website.
4. Developed the formula for the rotation by an angle ϕ about the origin, finally expressing it in matrix form.

22 Friday, October 12

1. Used the formula for rotation matrices to create these matrices for various angles. The matrix for a 0° rotation is the identity matrix, which makes sense. Noted that if you multiply the matrix for a 30° rotation by itself, you get the matrix for a 60° rotation, which makes sense.

If we denote the 30 degree rotation matrix by M_{30} , then we can see that the associative property of matrix multiplication shows that multiplying by two successive M_{30} matrices can be accomplished by multiplying by a single M_{60} matrix:

$$M_{30}(M_{30}P) = (M_{30}M_{30})P.$$

The matrix for a 90° rotation is

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

which makes sense, because if you rotate a point (x_1, y_1) by 90 degrees, its new coordinates are $(-y_1, x_1)$ (draw a picture). If you create the four matrices for multiplications by multiples of 90 degrees, M_0 , M_{90} , M_{180} , and M_{270} , then you can make a multiplication table. For example, $M_{90}M_{180} = M_{270}$, and $M_{270}M_{270} = M_{180}$.

2. Demonstrated the GeoGebra file, posted to the course website, that allows you to type in values for the matrix and visually see the result of multiplying the matrix by a point.
3. The above material concludes the material that will be on the next exam.
4. Thought about ways of making sense of the fact that multiplying a negative number by a negative number should be a positive number (using, e.g., positive and negative rates, and positive and negative time flow).
5. Introduced a way of thinking about multiplying positive and negative numbers—think of multiplying by a negative number as rotating 180 degrees counterclockwise and then multiplying by a positive number.
6. Extended the above to vectors in the plane. For each vector, determine its angle with respect to the x -axis, and its nonnegative length. Then to multiply two vectors, add the angles and multiply the lengths.
7. Concluded with comments, questions, and answers with a visitor: Dr. Wayne Nirode, who has been teaching for 16 years at Troy High School in Ohio, and who recently earned his Ph.D. in Mathematics Education.

23 Monday, October 15

1. Answered questions relating to the exam on Wednesday.
2. Discussed the geometrical interpretation of multiplying complex numbers: Represent them as points in the plane, use their polar coordinates, and then to multiply, add the angles and multiply the lengths. This allows us to make sense of such statements as $i^2 = -1$, and to determine solutions to equations like $z^3 = 1$. Also, we see that we divide complex numbers by subtracting the angles and dividing the lengths.

We justified this interpretation for multiplication by using the angle addition formulas.

The course notes for this discussion will be updated after Wednesday.

24 Wednesday, October 17

Exam #2.

25 Friday, October 19

1. Mentioned that two more motivations for “negative times negative equals positive” are the consistency of the distributive property, and Coulomb’s Law for electrical attraction.
2. Reviewed the geometric addition, subtraction, multiplication, and division of complex numbers.
3. Observed that if you have a complex number w with argument (angle) θ and modulus (length) r , then the function $f(z) = wz$ maps the entire complex plane to itself by rotating it by the angle θ counterclockwise about the origin and scaling it by a factor of r .
4. Demonstrated some of the above with GeoGebra.
5. Solved some problems like finding all three cube roots of a complex number (such as i).
6. Motivated the formula for the Taylor series of a function at $x = 0$ by the “guess my polynomial” games—determining a polynomial when all you are allowed to ask are questions of the form “What is $f^{(n)}(0)$?”

26 Monday, October 22

1. Returned exams.
2. Discussed Taylor series for the exponential, sine, and cosine functions—see Section 6.6 of the Course Notes.
3. Worked on ambigram puzzles as an introduction to transformations. I will post a link to the course website.
4. Watched a music video that demonstrates a clever superposition and repetition. Again, I will post a link to the course website.

27 Wednesday, October 24

1. Assigned Homework #6, also posted to the course website.
2. Handed out some more examples of ambigrams.
3. Practiced with translations, rotations, reflections, and glide reflections, using volunteers from the class. Observed the result of performing a succession of two reflections in two parallel lines appeared to be a translation, and in two crossing lines appeared to be a rotation.
4. Demonstrated some transformations with GeoGebra.
5. Worked on problem 7 of the new homework assignment with a “square dance.”. In particular, observed what the inverses were of some of these transformations.

28 Friday, October 26

1. Announcement of Martin Gardner Celebration today. The announcement is posted to the course website.
2. Explained the significance of the Smarter Balanced released items, with a link on the course website.
3. Mentioned Scott Kim's symmetry talk resources, with a link on the course website.
4. Introduced the National Library of Virtual Manipulatives. There are activities for all grade levels in all areas of math. I demonstrated briefly one on transformations, the virtual geoboard, and a few others. There is a link on the course website.
5. An isometry is a mapping of the plane to itself that preserves distances. It also preserves angle measure. Reviewed the four isometries of translation, rotation, reflection, and glide reflection—defining them and carrying them out given their elements. Translation and rotation are *direct* isometries, and reflections and glide reflections are *indirect* isometries.
6. Looked at some of the GeoGebra files on the course website, with the goal of identifying the isometry and determining its elements (e.g., vector of translations, line of reflection, center and angle of rotation, etc.). The trace feature of GeoGebra is very helpful in this respect. We saw how to find the center and angle for a rotation, and the line and vector of a glide reflection.

29 Monday, October 29

1. Discussed and worked on homework problems.
2. Handout: Isometry Problems I. We will look at some of these next time.

30 Wednesday, October 31

1. Discussed the homework problems, which I am willing to collect on Friday instead of today.
2. I will assign new homework shortly. The date for Exam #3 will be later than the tentative date in the syllabus.
3. Concerning the problem with the composition table, observed that the upper left hand portion represents a subgroup, and that it can also be interpreted as addition modulo 4. It may be helpful to construct this table with two copies of a square, with the corners labeled 1, 2, 3, and 4. Note also that order matters—reversing the order in which the operations are performed may have a different net result. Observed that this table can be constructed by using the formulas for each of the eight operations. For example, F_{45} maps (x, y) to (y, x) .
4. Concerning the problem with the formula for rotation about (p, q) , observed that this is an example of applying a conjugate. The net operation is $T^{-1}RT$, where T is the translation mapping (x, y) to $(x - p, y - q)$, and R is the rotation.
5. Briefly discussed the notion of congruence.

31 Friday, November 2

1. Worked on Isometry Problems I, Exercise 0.10.
2. Worked on Problems 7.2.10 and 7.2.11 in the Course Notes.

32 Monday, November 5

1. Discussed the theorem that every isometry of the plane is one of the four types we have encountered: translation, rotation, reflection, or glide reflection. We did not prove this.
2. Discussed Problem 7.2.12, illustrating this with GeoGebra. This is the theorem that an isometry is determined by its action on three noncollinear points. The location of the point $f(P)$ will be the intersection of three circles centered at $f(A)$, $f(B)$, and $f(C)$. Similar methods are used in surveying, in GPS location systems, and in determining the location and time of an earthquake.
3. Discussed Problem 7.2.13, illustrating this with GeoGebra. This is the theorem that every isometry is the result of applying (composing) at most three reflections. In particular, you should already know how to construct a pair of reflections to result in a given translation or rotation. In general, suppose you have three non-collinear points A, B, C mapping, respectively, to $\overline{A}, \overline{B}, \overline{C}$. If $A \neq \overline{A}$, construct a reflection across the perpendicular bisector of \overline{AA} to map A onto \overline{A} . Let the images of B and C be B' and C' . If $B' \neq \overline{B}$, construct a reflection across the perpendicular bisector of $\overline{B'B}$ to map B' onto \overline{B} . Observe that \overline{A} is on the reflection line, so will not move. Let the image of C' be C'' . If $C'' \neq \overline{C}$, construct a reflection across the perpendicular bisector of $\overline{C''C}$ to map C'' onto \overline{C} . Observe that \overline{A} and \overline{B} are on the reflection line, so will not move. The net effect of all of this is a isometry that maps A, B, C to $\overline{A}, \overline{B}, \overline{C}$, respectively.

Once you know this result, you can prove that every composition of three reflections is either a reflection or a glide reflection, and thus prove that every isometry is one of the four types we have discussed.

4. Discussed 3×3 matrices for translations. For a translation by (p, q) , use the matrix

$$\begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix}.$$

To apply to a point (x_1, y_1) , append a 1 to this point, write it as a column vector, and multiply

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

5. Discussed 3×3 matrices for rotations. To rotate by an angle ϕ about a point (p,q) , let $c = \cos \phi$ and $s = \sin \phi$. Use the matrix

$$\begin{bmatrix} c & -s & -cp + sq + p \\ s & c & -sp - cq + q \\ 0 & 0 & 1 \end{bmatrix}$$

To apply to a point (x_1, y_1) , append a 1 to this point, write it as a column vector, and multiply

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} c & -s & -cp + sq + p \\ s & c & -sp - cq + q \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

6. Reflections and glide reflections can similarly be expressed as 3×3 matrices of the form

$$\begin{bmatrix} c & s & u \\ x & -c & v \\ 0 & 0 & 1 \end{bmatrix}$$

where $s^2 + c^2 = 1$. (We did not prove this.)

7. One advantage to expressing isometries in matrix form is that we can apply a sequence of isometries given by matrices M_1 , M_2 , and M_3 to a point P by either computing $(M_3(M_2(M_1P)))$ or $(M_3M_2M_1)P$. The latter is convenient if we are applying these isometries to many points P , because we can compute $M = M_3M_2M_1$ and then multiply M by each of our points.

33 Wednesday, November 7

1. Briefly discussed the homework due next time.
2. Discussed some of the material on “Transforming” in the presentation <http://www.ms.uky.edu/~lee/arkansas2011/Functions.pdf>, slides 101–112, Also discussed how to determine the equation of an ellipse that has been rotated by 45 degrees—see slide 122 for a different rotation amount.

34 Friday, November 9

1. Worked on some GeoGebra modeling activities from <http://www.ms.uky.edu/~lee/arkansas2011/Functions.pdf>—see the section on Modeling. In particular, demonstrated using GeoGebra with sliders to control parameters for families of functions to model a fountain with a parabola and the St. Louis arch with a catenary. We also modeled circular motion, with a simplified model for a rotating planet moving around the sun.
2. Collected Homework.

35 Monday, November 12

1. Discussed 7.5.1–7.5.6.

36 Wednesday, November 14

1. Answered questions relating to the exam.

37 Friday, November 16

1. Exam #3.

38 Monday, November 19

1. Discussed some consequences of defining two figures being congruent if one can be mapped to the other by an isometry.
2. Sketched how this can be used to prove the SAS and the ASA triangle congruence criteria.

39 Monday, November 26

1. Returned the Exam #3.
2. Began discussing area. Starting from the formula for the area of a square, derived the formulas for the area of a rectangle and the area of a parallelogram.

40 Wednesday, November 28

1. The fourth and final exam will cover material on area, volume, surface area, and related material.
2. Worked on Problems 1–5 on Homework #8, due next Wednesday.

41 Friday, November 30

1. Reviewed how to use calculus to find areas and volumes by “slicing.” In particular, we found the area of a triangle and the volume of a right circular cone.
2. Described Cavalieri’s Principle for two-dimensional figures with cross sections having equal lengths with each other at all levels, and three-dimensional figures with cross sections having equal areas with each other at all levels.
3. Described two ways to motivate the formula for the area of a circle: one with pie-shaped pieces that are rearranged to form an approximation of a parallelogram or rectangle, one one with concentric circles unrolled to form a triangle. See the links for “Area of Circle” and “Unrolling a Circle” on the course website.
4. Began some work on Problems #6–10 of the homework.
5. Gave time at the end of class to fill out course evaluations.

42 Monday, December 3

1. Continued working on homework problems. The due date is now Friday, not Wednesday. Solutions will be posted on the course website on Friday or Saturday.
2. Used the SketchUp file “Scaling a Cube Building” on the course website to illustrate changes in volume and surface area when a “cube building” is scaled by a factor of three.

43 Wednesday, December 5

1. Demonstrated the planimeter, a mechanical device that measures area.
2. Worked on homework problems, due on Friday.

44 Friday, December 7

1. Collected the last homework assignment. When it is graded, it will be available in my mailbox in 715 Patterson Office Tower. I will send out an email about this. Homework solutions will be posted soon.
2. Solutions to most of the problems in the Area and Volume list are now posted on the course website.
3. Emphasized the importance of coupling math content (“knowing mathematics”) with math practices (“doing mathematics”).
4. Mentioned some websites, with links from the course website: Mathematics Education and Outreach Resources, On Being the Right Size, Flatland, Geometry in Utopia, Illustrative Mathematics.
5. Some general discussion about miscellaneous topics: Möbius strips, Klein bottles, Flatland, the fourth dimension and four-dimensional cubes, and visualizing mathematics, etc.