

**MA 341 Homework #6**  
**Due Wednesday, October 31, in Class**

1. On your own work the sample mathematics problems at the Smarter Balanced website <http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm>. These are sample problems that represent the kinds of problems being developed for tests used in many states nationally. You do not have to write up and submit your answers. Just write two or three sentences about your reactions to the questions involving geometry—Are they appropriate? Did you like them? Did you have trouble with any of them? Why or why not?
2. Find all solutions to  $z^4 = -81$ . Your answers should be exact, not approximations, and should not be left in the form of sines or cosines. Show your work.
3. Find one solution to  $z^3 = 4\sqrt{2} + (4\sqrt{2})i$ . Your answer should be exact, not an approximation, and should not be left in the form of sines or cosines. Hint: Use the half-angle formulas somewhere. Show your work.
4. Given two parallel lines  $\ell$  and  $m$  that are separated by a distance  $d$ . What is the net effect of first reflecting across  $\ell$  and then reflecting across  $m$ ? Be as specific as you can; in particular, you should make use of the number  $d$ . Justify your answer, including a clear diagram.
5. Given two distinct lines  $\ell$  and  $m$  that intersect at a point  $P$  such that the counterclockwise angle from  $\ell$  to  $m$  has measure  $\phi$ . What is the net effect of first reflecting across  $\ell$  and then reflecting across  $m$ ? Be as specific as you can; in particular, you should make use of the number  $\phi$ . Justify your answer, including a clear diagram.
6. Suppose that the point  $(x_1, y_1)$  is rotated counterclockwise by an angle  $\phi$  about the point  $(p, q)$ , resulting in the point  $(x_2, y_2)$ . Prove that

$$x_2 = cx_1 - sy_1 - cp + sq + p,$$

$$y_2 = sx_1 + cy_1 - sp - cq + q,$$

where  $c = \cos \phi$  and  $s = \sin \phi$ . Suggestion: First subtract  $(p, q)$  to translate the point  $(p, q)$  to the origin. Then perform a rotation about the origin. Then add  $(p, q)$  to undo the effects of the first translation.

7. Consider the following eight transformations:

- $I$ , the identity transformation.
- $R_{90}$ , rotation by 90 degrees counterclockwise about the origin.
- $R_{180}$ , rotation by 180 degrees counterclockwise about the origin.
- $R_{270}$ , rotation by 270 degrees counterclockwise about the origin.
- $F_0$ , reflection (“flip”) about the  $x$ -axis.
- $F_{45}$ , reflection about the line  $y = x$ .
- $F_{90}$ , reflection about the  $y$ -axis.
- $F_{135}$ , reflection about the line  $y = -x$ .

Fill in this multiplication table. The entry in row  $i$ , column  $j$  should be the net result of FIRST performing the transformation associated with COLUMN  $j$ , and THEN performing the transformation associated with ROW  $i$ . (You will have to redraw this table so that you have more room to write.)

| $\circ$   | $I$ | $R_{90}$ | $R_{180}$ | $R_{270}$ | $F_0$ | $F_{45}$ | $F_{90}$ | $F_{135}$ |
|-----------|-----|----------|-----------|-----------|-------|----------|----------|-----------|
| $I$       |     |          |           |           |       |          |          |           |
| $R_{90}$  |     |          |           |           |       |          |          |           |
| $R_{180}$ |     |          |           |           |       |          |          |           |
| $R_{270}$ |     |          |           |           |       |          |          |           |
| $F_0$     |     |          |           |           |       |          |          |           |
| $F_{45}$  |     |          |           |           |       |          |          |           |
| $F_{90}$  |     |          |           |           |       |          |          |           |
| $F_{135}$ |     |          |           |           |       |          |          |           |

8. EXTRA CREDIT. Create an ambigram based on your name, in the spirit of one of the “puzzle words” of the handout. So you should create a figure which, when superimposed with itself suitably, produces your first (or middle, or last, or first and last) name(s). Put some effort into a well-executed drawing.