MA 341 Homework #5

Due Friday, October 10, in Class

1. Consider the problem that we *already solved* using calculus.

   > A camper finds herself near (but not at) the bank of a straight river. Describe how to construct the shortest path from her current location to her tent, given that she wishes first to stop by the river. If the river bank is represented by the line $y = 0$, her present location by the point $A = (0, 2)$, and her campsite by the point $B = (6, 3)$, what is the shortest route she can take? Provide justification. Make a good sketch. It may be helpful to use GeoGebra to experiment.

   We figured out that we needed to find a particular point C on the river bank such that the angle that $\overline{AC}$ makes with the river matches the angle that $\overline{BC}$ makes with the river.

   Now solve the problem a different, essentially geometric way, by inserting the point $B' = (6, -3)$ into the diagram and thinking about various line segments.

2. A camper finds herself at a point $A(x_1, y_1)$ near (but not at) the bank of a straight river. Assume the closer bank of the river is given by the line $y = 0$. She can run at speed v and swim at speed w. She wants to get to a particular point $B(x_2, y_2)$ on the opposite bank of the river. So she runs to a point C on the near river bank and then swims from C to B. The water in the river is moving so slowly that during her swim you can neglect any movement downstream due to river flow. How can you determine the location of the point C that will minimize her total time?

3. A camper finds herself in the angle formed by the edge of a meadow and the bank of a river. Her tent is also in this angle. The bank of the river is given by line $y = 0$. The edge of the meadow is given by the line $y = x$. The camper is currently at the point $(9, 6)$, and the tent is at the point $(6, 3)$. Describe how to construct the shortest path from her current location to her tent, given that she wishes first to stop by the river, and then after that stop by the meadow, on the way to her tent. What is the shortest path from her current location to the river to the meadow to the tent?

4. Let $A = (-2, 0)$ and $B = (2, 0)$. Consider the set of all points $P = (x, y)$ such that the sum of the distances $PA + PB$ equals 6. Find an equation to describe this set of points, simplifying it as much as possible—in particular, figure out how to get rid of any square roots. Then use GeoGebra or a similar program to make a good sketch. What kind of shape do you get?

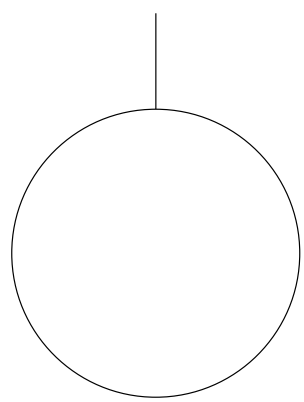
5. *It was the first time that Poole had seen a genuine horizon since he had come to Star City, and it was not quite as far away as he had expected. . . . He used to be good at mental arithmetic—a rare achievement even in his time, and probably much rarer now. The formula to give the horizon distance was a simple one: the square root of twice your height times the radius—the sort of thing you never forgot, even if you wanted to. . .*

—Arthur C. Clarke, *3001*, Ballantine Books, New York, 1997, page 71

In the above passage, Frank Poole uses a formula to determine the distance to the horizon given his height above the ground.

(a) Use algebraic notation to express the formula Poole is using.

(b) Beginning with the diagram below, derive your own formula. You will need to add some more elements to the diagram.



(c) Compare your formula to Poole's; you will find that they do not match. How are they different?

(d) When I was a boy it was possible to see the Atlantic Ocean from the peak of Mt. Washington in New Hampshire. This mountain is 6288 feet high. How far away is the horizon? Express your answer in miles. Assume that the radius of the Earth is 4000 miles. Use both your formula and Poole's formula and comment on the results. Why does Poole's formula work so well, even though it is not correct?