MA 341 Homework #2
Due Wednesday, February 5, in class)

1. Recall: A *point* in $\mathbf{E}^2$ is an ordered pair of real numbers (x, y), and a *line* in $\mathbf{E}^2$ is defined to be the set of points satisfying an equation of the form $ax+by+c=0$, where a and b are not both zero.

   Let $A+(x_1, y_1)$ and $B=(x_2, y_2)$ be two different points in the plane. Prove (using the above definition of a line):

   (a) $x(y_1-y_2)+y(x_2-x_1)+x_1y_2-x_2y_1=0$ is the equation of a line containing both A and B.

   (b) If $ax+by+c=0$, with a and b not both zero, is any linear equation satisfied by both A and B, then it must be a nonzero multiple of the above one.

2. Let S be a sphere in $\mathbf{R}^3$ of radius 1 centered at $O=(0,0,0)$. Let P be the plane $\{(x,y,z) : z=0\}$. Let $A=(x,y,z)$ be a point on S that is not equal to $N=(0,0,1)$. Let L be the line passing through A and N. Let $B=(u,v,0)$ be the point at the intersection of L and P.

   (a) Derive formulas for u and v in terms of x, y, and z.

   (b) Derive formulas for x, y, and z in terms of u and v.