The Deltahedra

In class we constructed the deltahedra. These are the convex three-dimensional polyhedra that are composed of equilateral triangles. We found deltahedra with 4, 6, 8, 10, 12, 14, 16, and 20 triangles, and these are all of them. It turns out that if we are allowed to use nonequilateral triangles as well, we can achieve any even number greater than or equal to 4. By looking at the dual graph, with one vertex for every triangle and one edge for every pair of neighboring triangles, we see that we have a graph in which every vertex has degree 3, which is an odd number. Since the number of odd degree vertices must be even, we conclude that we must use an even number of triangles.