MA/CS415 Homework #7
Due Wednesday, October 27

1. There is a polyhedron with the property that every face is a regular pentagon, square, or triangle, and the sequence of faces meeting at each vertex is triangle-square-pentagon-square (3,4,5,4). Use the method developed in class to determine the number of faces, the number of edges, and the number of polygons of each type.

2. Explain why there cannot be a polyhedron with the property that every face is a square, pentagon, or hexagon, and the sequence of faces meeting at each vertex is square-pentagon-hexagon (4,5,6).

3. Let P be a polyhedron with V vertices, E edges, and F faces.

   (a) Prove that $F \leq 2V - 4$.

   (b) Prove that $V \leq 2F - 4$.

   (c) Make a coordinate system with one axis for V and the other for F. Carefully sketch the above two inequalities, and mark the integer ordered pairs (V, F) that satisfy both inequalities. (GeoGebra is nice for this—choose View Grid and type in equations in the input line at the bottom.)

   (d) Make sketches of polyhedra for the following (V, F) points:

      i. (4,4)
      ii. (5,5)
      iii. (5,6)
      iv. (6,5)
      v. (6,6)
      vi. (6,7)
      vii. (7,6)