Geometry #4

Before Tuesday, September 11

Our goal this week is to consider the perhaps surprisingly deep notion of incidence of points and lines. Of course, "point", "line" and "incident" are undefined terms, so we will endow them with different meanings and take a look at the implications for some of the incidence axioms. Also, we may sometimes say that a point is "contained" in a line as another way of saying that the point is "incident" with the line, and that two lines "intersect" as another way of saying that there is at least one common point "incident" to both of them. In particular, we will try out a few models to see if they satisfy either, neither, or both of the following two axioms:

- Given any two different points there is exactly one line containing both of them.
- Given a point P and a line ℓ there is exactly one line m incident to P that does not intersect ℓ.

Read Dunham, Chapter 3, and read "Notes on Geometry" Chapter 6, available from the course website http://www.ms.uky.edu/~lee/ma501fa07/ma501fa07.html. Go to the Forum "Elements" and make at least one substantive contribution by 11 pm, Tuesday, September 11, and at least one substantive response to others' postings before class on Thursday, September 13. Dig into some of the later books of Euclid's *Elements*, taking advantage of the website http://aleph0.clarku.edu/~djoyce/java/elements/elements.html. In particular, take a look at some of the theorems concerning three-dimensional geometry. Then select a theorem in geometry in the *Elements* that appeals to you, study its proof, and write about why you made that choice, and where one is likely to encounter this result in the K–16 mathematics path. Please do not repeat someone else's choice!

Before Thursday, September 13

Read Dunham, Chapter 3, and read "Notes on Geometry" Chapter 6, available from the course website http://www.ms.uky.edu/~lee/ma501fa07/ma501fa07.html. As you read, think about the following questions for discussion.

1. What different topics do the various Books of Euclid's *Elements* address?
2. *Notes on Geometry*, Exercise 6.3.1 on page 32 applied to the models in Sections 6.3.1 through 6.3.8. Determine whether or not each model satisfies either, neither, or both of the following two axioms:

- Given any two different points there is exactly one line containing both of them.
- Given a point P and a line ℓ there is exactly one line m incident to P that does not intersect ℓ.

Thursday, September 13, 7–9 pm

Attend the Adobe Connect session to discuss the readings, discussion questions, forum, and comments and questions on the assigned homework due on Sunday.

Before Sunday, September 16, 11 pm

Homework problems due Sunday, September 16, 11 pm, uploaded to the Moodle site as a single file less than 2 MB, or else emailed to the address lee@ms.uky.edu. Please use Word or pdf files only.

1. *Notes on Geometry*, Exercise 6.3.1 on page 32. applied to the models in Sections 6.3.1 through 6.3.8. Determine whether or not each model satisfies either, neither, or both of the following two axioms:

   - Given any two different points there is exactly one line containing both of them.
   - Given a point P and a line ℓ there is exactly one line m incident to P that does not intersect ℓ.

2. Look up the game of "Set" (for example, http://en.wikipedia.org/wiki/Set_game). Can you come up with some sort of incidence axioms for points, lines, maybe planes, incidence, etc., so that the deck of Set cards provides a model? (My daughter sees the "sets" almost instantaneously—extremely intimidating!)

3. There are four regular solids known as the Kepler-Poinsot solids. What are they? Who discovered them and when? Why are they considered regular; i.e., why does it make sense to say that each face is the same regular polygon, and the same number of regular polygons meet at each vertex? What makes them different from the five classical regular solids, so that they were not discovered until many centuries later? If possible, include pictures—for PC users you can download the program Wingeom from http://math.exeter.edu/rparris/wingeom.html, start it up, open a 3-dim window, and go to Units→Polyhedral→Classics. Drawings from this program can be copied to clipboard from the "File" menu and then pasted into other documents. For Mac users (like me), I'm sure there are many images "out there."