Geometry #8

Before Tuesday, October 9, 11 pm

Go to the Forum "Resources" and discuss one website or piece of software that you find useful in the teaching and learning of geometry. Include the web address or information on where to get the software. (Try not to duplicate someone else's choice.) Then, by Thursday, October 11, 7 pm, take a look at a website or software that someone else has contributed and offer your comments about it.

Thursday, October 11, 7–9 pm

Attend the Adobe Connect session to discuss the forum, being prepared to show the class the website or software you have selected, and also to discuss comments and questions on the assigned homework due on Sunday.

Before Sunday, October 14, 11 pm

Homework problems due Sunday, October 14, 11 pm, uploaded to the Moodle site as a single file less than 2 MB, or else emailed to the address lee@ms.uky.edu. Please use Word or pdf files only.

1. Suppose we have a line ℓ and two distinct points A and B not on ℓ, and we want to find a point C on ℓ such that the sum of the distances $AC + CB$ is minimum. Let m be the line perpendicular to ℓ through C. Prove that for the point C that minimizes the sum $AC + CB$ it must be the case that the angle between AC and m, and the angle between BC and m, have the same measures. Easy suggestion: Let B' be the reflection of the point B in ℓ and consider the line segment $\overline{AB'}$. Harder suggestion: Use calculus.

2. Suppose we want to calculate the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) on a sphere of radius r centered at the origin by connecting them with the shortest arc that is part of a great circle. Prove that the distance is given by the formula
$$r \cos^{-1}\left(\frac{x_1x_2 + y_1y_2 + z_1z_2}{r^2}\right)$$
where $\cos^{-1}$ is the inverse cosine (i.e., arccos) function. You may use the last formula on page 99 of *Notes on Geometry* without proof, because its proof is essentially the same as the analogous formula in two dimensions, which you already verified. (Extra Credit: Prove algebraically that this formula satisfies the triangle inequality.)

3. Suppose we decided to define the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in three-dimensional space by the formula $|x_2 - x_1| + |y_2 - y_1| + |z_2 - z_1|$. Describe the shape of the set of all points having a distance of 1 from the origin.

4. Suppose we decided to define the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in three-dimensional space by the formula $\max\{|x_2 - x_1|, |y_2 - y_1|, |z_2 - z_1|\}$. Describe the shape of the set of all points having a distance of 1 from the origin.