MA 501 — Log of Class Activities

Contents

1 Tuesday, January 15

1. Members of the class introduced themselves.

2. Distributed the syllabus, also on the course website—class members had been asked to review this in advance.

3. Polydron activity. Everyone generated names of two-dimensional figures. There was a short discussion whether a kite could be concave (sometimes called a "dart"). The page in Wikipedia, http://en.wikipedia.org/wiki/Kite_%28geometry%29, states that kites can be concave, but sometimes this word is used only for the convex case.

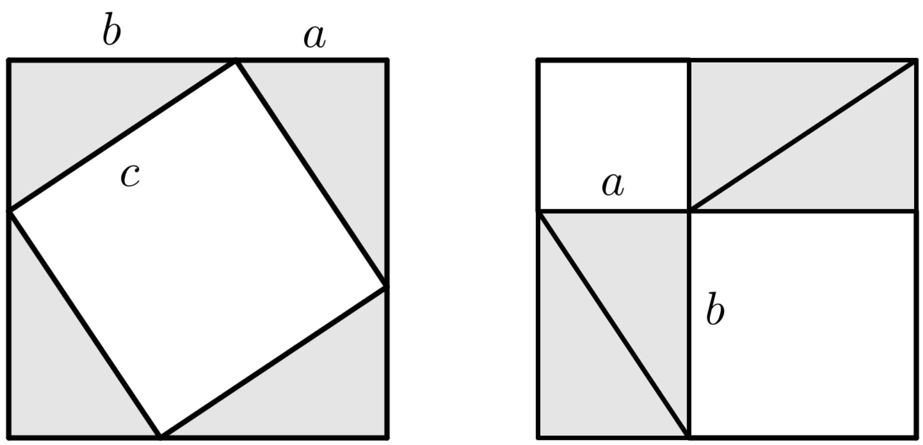
   In the next step three-dimensional shapes were constructed with the manipulative Polydron, with the goal of creating "analogs" of some of the two-dimensional figures. For example, the tetrahedron can be regarded as an analog of a triangle because the triangle is the polygon with the fewest number of vertices, and the tetrahedron is the polyhedron with the fewest number of vertices. The cube can be regarded as an analog of a square, because it consists of pairs of parallel opposite faces, with faces meeting along edges at right angles, and all edge lengths being equal. We noted that if you tore the four corners off of a square they could be assembled to fit together around a point in the plane, and likewise if you cut off the eight corners of a cube they could be assembled to fit together around a point in space. The sphere is an analog of the circle, and the parallelepiped is the analog of a parallelogram—see http://en.wikipedia.org/wiki/Parallelepiped. The regular tetrahedron, the cube, and the dodecahedron are analogs of regular polygons, in that every face is congruent to a common regular polygon, and the solid angle is the same at each vertex—the same number of faces meet at each vertex. Such regular convex polyhedra are known as the Platonic solids; see http://en.wikipedia.org/wiki/Platonic_solid.

   We also constructed prisms http://en.wikipedia.org/wiki/Prism_%28geometry%29, antiprisms http://en.wikipedia.org/wiki/Antiprism, and some others, such as the truncated tetrahedron http://en.wikipedia.org/wiki/Truncated_tetrahedron. Note that a soccer ball is a truncated icosahedron, http://en.wikipedia.org/wiki/Truncated_icosahedron.

4. I mentioned that all of the high school geometry books in the SMSG series, including the teacher guides, are now available free online; see http://ceure.buffalostate.edu/~newmath/SMSG/SMSGTEXTS.html. I have posted a "summary" here: http://www.ms.uky.edu/~lee/ma341fa12/smsg.pdf. (I used some of these texts when I was in high school.)

5. Here is my list of mathematics education and outreach resources: `http://www.ms.uky.edu/~lee/outreach/outreach.html`. I hope we can discuss and share some of these, and that you will have others to add. Note in particular that one item on this website is a list of potentially useful iPad apps.

6. Introduced SketchUp, `http://www.sketchup.com`. I used SketchUp with a geometry class of eight grade students at Jessie Clark Middle School in Lexington; see `http://www.ms.uky.edu/~lee/jessieclark/jessieclark.html`. Passed out a copy of a brief tutorial that I used for this project: `http://www.ms.uky.edu/~lee/jessieclark/NCTMDenver.pdf`. Demonstrated how to make some shapes (e.g., prism, cylinder, sphere, torus). Class members practiced with some of these constructions. I started making a diagram to represent a proof of the Pythagorean Theorem. Here is the completed diagram, but made with GeoGebra:



Without algebra you can see that the two large square frames each contain four copies of the right triangle under consideration, so the areas inside the frames but outside the triangles must be the same. In the first case this area is c^2, and in the second is $a^2 + b^2$. So these two quantities are the same. With algebra, you can use just the first frame to observe that $(a + b)^2 = 4 \cdot \frac{1}{2}ab + c^2$. (The right hand side is the area of the four triangles plus the area of the square.) This simplifies to $a^2 + b^2 = c^2$.

7. Briefly introduced GeoGebra, `http://www.geogebra.org`. Demonstrated constructing secant lines to parabolas, the intersection of the perpendicular bisectors for a trian-

gle (which is the center of the circumcircle), and using sliders to demonstrate shifting functions. I will put these GeoGebra files on the course website.

2 Tuesday, January 22

1. Discussion on Homework—SketchUp files, and questions.
2. Discussion on Homework—CCSSM standards. For a some exercises leading to the Law of Sines and the Law of Cosines, see Section 3,7 of `http://www.ms.uky.edu/~lee/ma241/ma241notesb.pdf`.
3. Discussion on Homework—Perpendicular bisectors.
4. Demonstrated using SketchUp for some disssections for area formulas.
5. Demonstrated making nets in SketchUp—unfolding polyhedra.

3 Tuesday, January 29

1. Discussion on Homework—some ways to approach the Rotating Square problem. In particular, symmetry (by placing four copies of the rotated square on top of the base square) can make the 1/4 area result much clearer.

2. Discussion of some GeoGebra constructions and some Illuminationts activites. In particular, one class member posted the sketch to geogebratube, which is a nice way to make it available to students and teachers.

3. Worked on the problem of identifying symmetries in various images, `http://www.ms.uky.edu/~lee/ma501sp13/symmetryslides2.pdf`. This leads in a natural way to thinking about types of isometries—rotations, reflections, translations, and glide reflections. See the initial sections of Symmetries and Transformations, `http://www.ms.uky.edu/~lee/ma501sp13/trans.pdf`. We also looked at Scott Kim's puzzles, `http://scottkim.com/thinkinggames/nctm2011handout.pdf`, the iOrnament app, `https://itunes.apple.com/us/app/iornament-the-art-of-symmetry/id534529876?mt=8`, and mentioned a website to generate graph paper, `http://incompetech.com/graphpaper`. Scott Kim has a nice set of resources on symmetry from an NCTM talk, `http://scottkim.com/thinkinggames/nctm2011.html`.

4 Tuesday, February 5

1. Discussed changing the format of the class during public school spring break, April 1–5. I will have to think about this. I will also think about what to do about the March 26 Career Fair at EKU.

2. Showed the figure to demonstrate the Law of Cosines in the case that the triangle is acute—`http://www.cut-the-knot.org/pythagoras/DonMcConnell.shtml`. It would be easy to turn this into a GeoGebra sketch. It would require a bit of thought to adapt it to obtuse triangles, but should not be too difficult.

3. Mentioned the Symmetry and Escher website for a full college course on this topic, `http://euler.slu.edu/escher/index.php/Course:SLU_MATH_124:_Math_and_Escher_-_Fall_2008_-_Dr._Anneke_Bart`.

4. Gave "proof without words" demonstrations of two summation formulas: (a) the sum of the first n positive integers is $\frac{n(n+1)}{2}$, and (b) the sum of the first n positive odd numbers is n^2. You can probably find these and many others by searching for "proof without words" on the web.

5. Practiced with Miras. These are helpful for both identifying reflective symmetry in a given figure, and in constructing the reflection of a given figure.

6. Practiced with whole body reflections, using meter sticks as mirrors. In particular, we looked at a reflection in a single line, in a sequence of reflections across two parallel lines, and in a sequence of reflections across two perpendicular lines.

   This led to a conjecture that if lines ℓ_1 and ℓ_2 are parallel, then the net effect of reflecting across ℓ_1 and then reflecting the result across ℓ_2 is a translation by an amount equal to twice the distance between ℓ_1 and ℓ_2 in a direction from ℓ_1 towards ℓ_2 perpendicular to both lines. We constructed this in GeoGebra, and by adding some auxiliary line segments were able to provide a proof in several cases. I will post the GeoGebra file on the course website.

   The net effect of a sequence of reflections across two perpendicular lines was observed to be a 180 degree rotation about their intersection point.

7. Practiced with the GeoGebra applets on the course website Isometry 1 through Isometry 8. Isometries 1 through 4 have the advantage that you can determine the isometries from the static diagram, or from a printout of the diagram. But this is not the case for Isometries 5 through 8—what happens to only one point is not sufficient to determine

the isometry. In fact, you need to know what the effect is on at least three noncollinear points.

We saw and argued that the center of rotation lies on the perpendicular bisectors of segments $\overline{PP'}$, where P is a point and P' is its image under the action of the rotation. Though we did not mention it, once you have the center of rotation, the angle of rotation is easy to construct. If you have a reflection, the line of reflection is the perpendicular bisector of each segment $\overline{PP'}$. If you have a glide reflection, the line of glide reflection passes through the midpoints of all of the segments $\overline{PP'}$, but is not perpendicular to them (unless the "translation part" of the glide reflection is zero). If you have a translation, the vector of translation points from any point P to its image P'.

All of these exercises help prepare for the next round of tasks—understanding the isometries of the strip.

8. Worked on Homework #4, which will be collected next time.

5 Tuesday, February 12

1. Reviewed the sequence of concepts that we are working through with respect to rigid motions. There are rich possibilities for informal and formal reasoning, use of technology to demonstrate and to providde rich tasks, to link geometry with algebra, and to see applications to real life (symmetries used by various cultures).

2. Reviwed the answers to nearly all of the homework problems and collected Homework #4.

3. Developed the algebraic formulas for the various strip isometries, and then used them to identify and to compose symmetries, which in retrospect provided another way to solve the homework problems that was analytic rather than synthetic.

4. Worked through the reasoning for why there are only seven possible different types of symmetry for repeating border patterns, relying on knowledge of the results of composing various symmetries.

5. Passed out a handout on Trigonometry, also posted to the course website.

6 Tuesday, February 19

1. Discussed the "Trigonometry" handout, most of Sections 1–2, and the beginning of Section 3. We stopped at Problem 3.7, and we did not review the proof of the Law of Cosines. See also the presentation "Functions" posted on the course website, which is a good outline.

2. Discussed the homework; in particular, we looked at ways to approach "solving triangles" with the Law of Sines and the Law of Cosines, including cases in which there were no solutions or more than one solution. GeoGebra was used to make some sketches to solve triangles; the one for SSA is posted on the course website.

3. Collected homework.

7 Tuesday, February 26

1. Worked through Trigonometry, sections 2 and 3, skipping cylindrical and polar coordinates.

2. Discussed homework on complex numbers.

8 Tuesday, March 5

1. Geometry of Circles, Cassie Atkinson, Michelle Ehme, and Holly Lawrence. Notes: This is a case where it's better to use the Miras than technology. This also lends itself to "patty paper" constructions.

2. Inequalities in Triangles, Rachel Crawford and Samantha Rogers. Notes: What is the probability of being able to make a triangle if the breaks are made at random locations? Consider using flat pasta.

3. Pi Line, Jessica Doering, Scott Emmons, and Kate Johnson. Notes: A link between geometry and algebra (graphing proportional quantities. GeoGebra could be used, including for regression.

4. Linear Alignment, Sharon Bixler. Notes: One could also use GeoGebra.

5. Adding it all Up, Renee Mooney. Notes: This offers an opportunity to make a good guess, and then justify it with a geometric argument. Nonconvex polygons lead to some interesting considerations.

9 Tuesday, March 19

1. Cutting Conics, Bryley Murphy. Notes: This appears to be a very useful applet. Mentioned how to find the foci of the ellipse within the cut cone, see http://en.wikipedia.org/wiki/Dandelin_spheres; curve stitching the parabola, ellipse, and hyperbola, see http://www.ms.uky.edu/~lee/ma501sp13/parabolastitch.ggb, http://www.ms.uky.edu/~lee/ma501sp13/ellipsestitch2.ggb, http://www.ms.uky.edu/~lee/ma501sp13/ellipsestitch.ggb; describing the conic sections as sets of points satisfying certain distance conditions, see http://www.cut-the-knot.org/Curriculum/Geometry/TwoCircleFamilies.shtml, http://www.ms.uky.edu/~lee/ma501sp13/parabolalocus.ggb, http://www.ms.uky.edu/~lee/ma501sp13/ellipselocus.ggb, http://www.ms.uky.edu/~lee/ma501sp13/hyperbolalocus.ggb, and paper folding the parabola, http://www.cut-the-knot.org/Curriculum/Geometry/ParabolaEnvelope.shtml.

2. Pinwheel, Maranda Miller. Notes: There are lots of activities bringing together math and origami. Examples: unit origami for two- and three-dimensional structures, see http://www.origamee.net/diagrams/diagrams.html; flexagons, see http://www.youtube.com/watch?v=VIVIegSt81k, http://www.youtube.com/watch?v=paQ10POrZh8.

3. Corner to Corner, Jamie-Marie Wilder. Notes: An approach to introducing the Pythagorean Theorem. The "Shape Cutter" applet helped visualize approximate values of irrational lengths.

4. We also watched the Vi Hart video "Infinity Elephants," http://www.youtube.com/watch?v=DK5Z709J2eo.

5. Began a look at the transformational approach to congruence, The main idea is that two figures (subsets of the plane) are congruent if one can be mapped to the other by a sequence of rigid motions (isometries). sketching the proof of SAS. Start looking at the middle and high school sections of congruence in the book http://math.berkeley.edu/~wu/Progressions_Geometry.pdf to compare with the more traditional approach of the SMSG series.

10 Tuesday, March 26

1. Demonstrated some GeoGebra files relating to conic sections, all posted on the course website: Parabola Locus, Parabola Curve Stitching, Ellipse Locs, Hyperbola Locus, Ellipse Curve Stitching, and Ellipse and Hyperbola Curve Stitching.

2. Looked at Euclid's "proof" of SAS—Euclid's Elements are nicely available online at `http://aleph0.clarku.edu/~djoyce/java/elements/elements.html`. See Book I, Proposition Proposition 4.

3. Looked at three proofs of the theorem that if a triangle is isosceles (has a pair of congruent sides) then it has a pair of congruent (base) angles—SMSG high school text, Euclid I:5, and via rigid motions (in this case, a reflection).

11 Tuesday, April 2

1. Students worked individually on the reading and problem assignments.

12 Tuesday, April 9

1. Presentations on proofs connected to congruence. This served as an example of a short strand in the axiomatic development of results—we used the sequence of results laid out in SMSG to avoid circular reasoning. We also discussed some changes that could be made in light of a stronger, earlier emphasis on transformations.

2. Discussed problem #2a of Homework #9.

13 Tuesday, April 16

1. Returned and discussed homework.

2. Discussed new homework.

3. Activity on areas of irregular shapes using grids of different sizes, with over and underestimates. Mentioned visualizing unit conversion. Handout: Example from Connected Mathematics.

4. Area of infinitely long shape—rectangles with base 1 and height $(1/2)^n$. This cannot be handled by grids in the same way, but can be handled by an infinite converging sum.

5. Snowflake curve—another shape whose area can be handled by an infinite sum. The area is finite but the perimeter is infinite.

6. Constructed 2D figures with Polydron squares and 3D figures with Multilink cubes. Scaled in one or more directions by one or more different amounts to visualize the effects of scaling.

7. Used Polydron to construct a dissection of a prism into three equal volume pyramids, to demonstrate the formula for the volume of a pyramid.

14 Tuesday, April 23

1. Attempted to construct all possible convex polyhedra with equilateral triangles (using Polydron). These are the deltahedra. We proved by a special case of the Handshaking Theorem that these objects must have an even number of triangles.

2. Constructed a collection of semiregular polyhedra with Polydron. These have the properties that every face is a regular polygon, that there are at least two different types of regular polygons, and that the same cyclic sequence of polygons occurs at each vertex. There are two infinite families of prisms and antiprisms, and apart from these there are thirteen Archimedean solids.

3. Partially constructed the crystal structure for diamond with Zome. See `http://en.wikipedia.org/wiki/Diamond_cubic`.

4. Described some of the resources listed in my Mathematics and Education Outreach site, `http://www.ms.uky.edu/~lee/outreach/outreach.html`.