MA 501 Homework #6

Due Tuesday, February 26, in class

This homework uses GeoGebra sketches posted on the course website.

1. Visualizing Complex Numbers. Open the sketch "Coordinates." The point z represents a complex number. You can see its value in the Algebra window. The length of the vector Az is the "modulus" of z. The measure of the angle $\angle BAz$ is the "argument" of z. Use the Move tool to drag the point z around, watching how its value, modulus, and argument change. Then find the answers to the following questions.

   (a) How do the x and y coordinates of the point z relate in a very simple way to its value as a complex number?

   (b) What can you say about complex numbers on the x-axis? On the y-axis?

   (c) Find the modulus and argument of the complex number $1 - i$.

   (d) What complex number has modulus 2 and argument 60 degrees?

   (e) The "conjugate" of a complex number $z = a+bi$ is the complex number $\overline{z} = a-bi$. How are z and $\overline{z}$ related geometrically?

2. Adding Complex Numbers. Open the sketch "Adding." The points z_1 and z_2 represent complex numbers, and the point w represents their sum. You can see their values in the Algebra window. Use the Move tool to drag the points z_1 and z_2 around. Then find the answers to the following questions.

   (a) How can you find the sum $w = z_1 + z_2$ algebraically?

   (b) How can you find the sum $w = z_1 + z_2$ geometrically? What additional things could you draw in the Graphics window to make this clearer?

   (c) How can you find the difference $w - z_2$ geometrically?

   (d) If you fix the complex number z_1 (don't move it), and move around the point z_2, what is the relationship between the points z_2 and w? Express your answer in the language of rigid motions.

3. Multiplying Complex Numbers. Open the sketch "Multiplying." The points z_1 and z_2 represent complex numbers, and the point w represents their product. You can see their values in the Algebra window, as well as their moduli (lengths) and arguments (angles). Use the Move tool to drag the points z_1 and z_2 around. Then find the answers to the following questions.

(a) What is the relationship of the argument of w to the arguments of z_1 and z_2?

(b) What is the relationship of the modulus of w to the moduli of z_1 and z_2?

(c) What is $(0+i)(0+i)$?

(d) Find two complex numbers whose product is $0+i$.

(e) If you fix the complex number z_1 to be $0+i$ (don't move it), and move around the point z_2, what is the relationship between the points z_2 and w? Express your answer in the language of rigid motions.

(f) If you fix the complex number z_1 to be $0+2i$ (don't move it), and move around the point z_2, what is the relationship between the points z_2 and w? Express your answer in the language of rigid motions.

(g) In general, if you fix the complex number z_1 (don't move it), and move around the point z_2, what is the relationship between the points z_2 and w? Express your answer in the language of rigid motions, explicitly describing the roles of the argument and modulus of z_1.

(h) If you multiply a complex number z by itself, describe the relationship of the argument and modulus of the result to those of z.

(i) Describe the relationship of the argument and modulus of z^3 to those of z.

(j) Find two different solutions to $z^2 = -1 + 0i$.

(k) How can you find the argument and the modulus of the quotient w/z_2 from those of w and z_2?

(l) What are all solutions to $z^3 = 1 + 0i$?

(m) What are all solutions to $z^4 = 0 + 16i$?

4. Iterations. Open the sketch "Iteration." The points A_1 through A_{20} represent complex numbers determined in the following way: A_1 is freely chosen, and each subsequent A_i equals $A_{i-1}^2 + A_1$.

   (a) Use the Move tool to move around A_1. Try to describe the nature of the various results. Don't worry about being precise mathematically; just talk about different phenomena you observe.