

MA 501 Homework #8

Due Tuesday, March 19, in class

The Common Core State Standards do not encourage a tight and complete axiomatic development of plane geometry. Nevertheless, geometric results can be organized and unfolded from the simple to the more complex, starting with a set of assumed axioms (or postulates), and this is one of the characteristics of *Euclid's Elements*, one of the most influential books in human history. (For an online version, see <http://aleph0.clarku.edu/~djoyce/java/elements/elements.html>.) The purpose of this homework assignment is to look at some short episodes in this sort of development.

My high school geometry text, together with the accompanying teacher guides, are online at <http://ceure.buffalostate.edu/~newmath/SMSG/SMSGTEXTS.html>—see Units 13–16. Please download these, allowing sufficient time to do this since they are rather large pdf files. I have prepared a summary, giving the sequence of results, but no proofs; see <http://www.ms.uky.edu/~lee/ma341fa12/smsg.pdf>. The idea is that any theorem in this list can be proved using only the preceding results.

Study the proofs of the following theorems (some of the proofs are to be found in the Teacher's Commentary). You do not need to write anything up to hand in, but I will be asking for some of these to be presented and discussed in class, so be prepared to be asked to present these on the board.

1. Theorem 5-2. If two sides of a triangle are congruent, then the angles opposite these sides are congruent.
2. Theorem 5-4. (The A.S.A. Theorem.) Given a correspondence between two triangles, (or between a triangle and itself). If two angles and the included side of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.
3. Theorem 5-5. If two angles of a triangle are congruent, the sides opposite these angles are congruent.
4. Theorem 5-6. (The S.S.S. Theorem.) Given a correspondence between two triangles (or between a triangle and itself). If all three pairs of corresponding sides are congruent, then the correspondence is a congruence.
5. Theorem 7-1. (The Exterior Angle Theorem.) An exterior angle of a triangle is larger than either remote interior angle.

6. Theorem 7-2. (The S.A.A. Theorem.) Given a correspondence between two triangles. If two angles and a side opposite one of them in one triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence. (Note that at this point it is not yet known that the sum of the measures of the angles of a triangle is 180.)
7. Theorem 7-3. (The Hypotenuse-Leg Theorem.) Given a correspondence between two right triangles. If the hypotenuse and one leg of one triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.
8. Theorem 9-13. The sum of the measures of the angles of a triangle is 180.
9. Corollary 9-13-3. For any triangle, the measure of an exterior angle is the sum of the measures of the two remote interior angles.
10. Theorem 9-14. Either diagonal separates a parallelogram into two congruent triangles. That is, if $ABCD$ is a parallelogram, then $\triangle ABC \cong \triangle CDA$.
11. Theorem 9-15. In a parallelogram, any two opposite sides are congruent.
12. Theorem 9-16. In a parallelogram, any two opposite angles are congruent.
13. Theorem 9-17. In a parallelogram, any two consecutive angles are supplementary.
14. Theorem 9-18. The diagonals of a parallelogram bisect each other.
15. Theorem 9-19. Given a quadrilateral in which both pairs of opposite sides are congruent. Then the quadrilateral is a parallelogram.
16. Theorem 9-20. If two sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.
17. Theorem 9-21. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
18. Theorem 9-22. The segment between the mid-points of two sides of a triangle is parallel to the third side and half as long as the third side.
19. Theorem 9-23. If a parallelogram has one right angle, then it has four right angles, and the parallelogram is a rectangle.
20. Theorem 9-24. In a rhombus, the diagonals are perpendicular to one another.

21. Theorem 9-25. If the diagonals of a quadrilateral bisect each other and are perpendicular, then the quadrilateral is a rhombus .
22. Theorem 11-7. (The Pythagorean Theorem). In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.
23. Theorem 11-8. If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle, with a right angle opposite the first side.