**Syllabus for Topics in Geometry**
**Summer 2004**

**Course:** Topics in Geometry

**Instructor:** Carl Lee

**Email:** lee@ms.uky.edu

**Webpage:** www.ms.uky.edu/~lee

**Course Webpage:** www.ms.uky.edu/~lee/ma501su04/ma501su04.html

**Prerequisites:** An undergraduate geometry course and some elementary calculus.

**Text:** David Kay, *College Geometry: A Discovery Approach*, second edition, Pearson Addison Wesley, ISBN: 0321046242.

**Grading:** Your grade will be determined by your achievements on a set of classroom activities and homework assignments.

There will be a variety of assignments: some will be short answers to questions, others will be longer solutions to more difficult problems, still others will be reflections on a larger body or sequence of material. Nearly all will be intertwined with the classroom activities. It will be necessary for students to maintain a comprehensive notebook of course notes and assignments. I suggest a loose-leaf notebook into which material can be easily extracted and inserted.

**Working Together:** It is ok to work together on homework. However, when it comes time for you to write up the solutions, I expect you to do this on your own, and it would be best for your own understanding if you put aside your notes from the discussions with your classmates and wrote up the solutions entirely from scratch.

## Course Content

The specific topics I plan to address are:

1. Geometrical Overview/Review
2. Axiomatic Systems
3. Points, Lines and Incidence
4. Coordinates
5. Distance
6. Angles
7. Area and Volume
8. Polyhedra
9. Congruence and Symmetry
10. Dimension
11. Euclidean and Non-Euclidean Geometry

David Hilbert wrote the following in 1932 in the introduction to his book with S. Cohn-Vossen, *Geometry and the Imagination*, and this has been a significant influence on my feelings towards geometry.

> In mathematics, as in any scientific research, we find two tendencies present. On the one hand, the tendency toward *abstraction* seeks to crystallize the *logical* relations inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency toward *intuitive understanding* fosters a more immediate grasp of the objects one studies, a live *rapport* with them, so to speak, which stresses the concrete meaning of their relations.

As to geometry, in particular, the abstract tendency has here led to the magnificent systematic theories of Algebraic Geometry, of Riemannian Geometry, and of Topology; these theories make extensive use of abstract reasoning and symbolic calculation in the sense of algebra. Notwithstanding this, it is still as true today as it ever was that *intuitive* understanding plays a major role in geometry. And such concrete intuition is of great value not only for the research worker, but also for anyone who wishes to study and appreciate the results of research in geometry.

In this book, it is our purpose to give a presentation of geometry, as it stands today, in its visual, intuitive aspects. With the aid of visual imagination we can illuminate the manifold facts and problems of geometry, and beyond this, it is possible in many cases to depict the geometric outline of the methods of investigation and proof, without necessarily entering into the details connected with the strict definitions of concepts and with the actual calculations. For example, the proof of the fact that a sphere with a hole can always be bent—no matter how small the hole—or of the fact that two different toroidal surfaces can not in general be wrapped onto each other conformally, can be treated in such a fashion that even one who does not wish to follow the details of the analytical arguments, may still gain an insight into how and why the proof works.

In this manner, geometry being as many-faceted as it is and being related to the most diverse branches of mathematics, we may even obtain a summarizing survey of mathematics as a whole, and a valid idea of the variety of its problems and the wealth of ideas it contains. Thus a presentation of geometry in large brush-strokes, so to speak, and based on the approach through visual intuition, should contribute to a more just appreciation of mathematics by a wider range of people than just the specialists. For it is true, generally speaking, that mathematics is not a popular subject, even though its importance may be generally conceded. The reason for this is to be found in the common superstition that mathematics is but a continuation, a further development, of the fine art of arithmetic, of juggling with numbers. Our book aims to combat that superstition, by offering, instead of formulas, figures that may be looked at and that may easily be supplemented by models which the reader can construct. This book was written to bring about a greater enjoyment of mathematics, by making it easier for the reader to penetrate to the essence of mathematics without having to weight himself down under a laborious course of studies.

The following description comes from the proposal for a new course in geometry for future middle school teachers at UK, and reflects the approach and attitude I will attempt to take with our present course.

**Overview:** Kentucky's teacher education institutions lack a course in geometry and measurement specifically designed to provide prospective middle school teachers with the mathematics background required in these areas to teach by state and national standards. The proposed course will be designed to increase the depth of the students' understanding in (a) the geometry and measurement standards identified in the *Kentucky Mathematics Core Content for Assessment for Grade 8* and in the *Kentucky Program of Studies for Grades 6-12*, and (b) the Geometry and Measurement Standards of the *NCTM (2000) Principles and Standards for School Mathematics, Grades 6-12.* Teaching and learning in this course will model the NCTM Process Standards: Problem-Solving, Communications, Connections, Reasoning and Proof and Representations. The content of the course will also align with the geometry and measurement items of the Middle School Mathematics Content Test of *PRAXIS II* (passing score required to obtain teaching certification in Kentucky).

**Objectives:** The course has three main objectives: (1) Increase the students' awareness and understanding of the scope and nature of geometry, including recent developments and applications, and connections to nature and art. (2) Follow selected fundamental themes and concepts in geometry as they are developed in the middle school, high school, and college curriculum. (3) Approach geometry in an investigative manner, using such techniques as collaborative learning; exploration and problem solving to formulate, test, and prove or disprove conjectures; and written and oral assignments to develop effective communication skills; and such tools as physical manipulatives; models; and software.

**Nature and Scope of Geometry:** Despite many developments in geometry (some quite recent), such as aperiodic tilings, graph theory, computational geometry, convex polyhedra and mathematical programming, discrete geometry, and solid modeling, the main focus of most pre-college and college geometry courses in Kentucky is still the axiomatic method as applied to two-dimensional Euclidean geometry. Most teachers have had little or no exposure to anything else, nor are encouraged to alter their curriculum they are asked to teach. This is completely contrary to recommendations such as those articulated, for example, in *Heeding the Call for Change*: "Mathematics departments should encourage prospective teachers to be exposed to both the depth and breadth of geometry." Course topics will be selected to draw the students from their more traditional background in geometry into its modern manifestations.

**Development of Themes:** There still appears to be a tendency for prospective teachers to wish to dismiss geometry above the level at which they intend to teach as largely irrelevant to their intended profession. More care must be taken to demonstrate how certain geometric themes or topics that they will be teaching to their middle school students will reappear and be articulated in greater depth and sophistication as their students leave them to advance through high school and college. One motivation for teachers to master their subject beyond their level of teaching is to be consciously aware of what they are preparing their students to meet in the future. The *NCTM Standards* provide good guidelines for this development, and *Heeding the Call for Change* encourages, “More emphasis should be placed on central conceptual aspects of geometry, such as geometric transformations and their effects on point sets, distance concepts, surface concepts, etc.” Topics in the course can be introduced using, for example, some of the modules in the NSF-funded middle school curricula: *Connected Mathematics Project*, *MATHThematics*, *Mathematics in Context*, and *Mathscape*. Then the topic can be revisited from more advanced perspectives, incorporating high school activities, college-level analysis, and applications. Some specific themes to be addressed are: one- two- and three-dimensional shapes; spherical figures; coordinates and analytical geometry; distance and length; perimeter, area, surface area, and volume; measurement and approximation; congruence and similarity; transformations; axioms and deduction. Of course, these topics are deeply intertwined, and it will not be possible to treat any one of them completely and comprehensively, but this interconnectedness will be reflected in the course work, and students will be exposed to important elements of each theme.

**Investigative Approach:** In consonance with the *NCTM Standards*, *Heeding the Call for Change* makes recommendations on methods of presentation that will be incorporated into the geometry course: “Geometric objects and concepts should be studied more from an experimental and inductive point of view rather than from an axiomatic point of view.” Resulting conjectures can then be proved or disproved. “More use of diagrams and physical models as aids to conceptual development in geometry should be explored.” Many geometry manipulatives (such as Polydron) are still rather expensive, but two-and three- dimensional models can be constructed from less costly materials. “Group learning methods, writing assignments, and projects should become an integral part of the format in which geometry is taught.” The NSF-funded curricula are currently being used in some middle schools throughout Kentucky. Therefore, collaborating middle school teachers can provide examples of students’ responses to aspects of the lessons (e.g., students’ work, journals, projects, scripts of classroom discourse, and videotapes of students small group discussions). Such examples will give students in the course an opportunity to discuss and analyze the thinking and concept development of “real middle school students.”

**Taking Advantage of Technology:** *Heeding the Call for Change* also emphasizes that a "wide variety of computer environments should be explored ... both as exploratory tools and for concept development." AMATYC, addressing general technology, declares, "The technology must have graphics, computer algebra, spreadsheet, interactive geometry, and statistical capabilities." For this geometry course, both geometry-specific software, such as Geometer's Sketchpad, and computer algebra software with graphics, such as Maple, can be used effectively for exploration and modeling.

**References:**


1. *Connected Mathematics Project*, http://www.mth.msu.edu/cmp.
2. *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*, AMATYC, 1995.
3. The Geometer's Sketchpad, http://www.keypress.com/sketchpad.
4. *Mathematics in Context*, http://www.edc.org/mcc/cmic.htm.
5. *Mathscape*, http://www.edc.org/mcc/cscape.htm.
6. *MATHThematics*, http://www.edc.org/mcc/cstem.htm.
7. *The Praxis Series: Professional Assessments for Beginning Teachers*, ETS, http://www.teachingandle
8. *Principles and Standards for School Mathematics*, NCTM, 2000, http://www.nctm.org/standards.
9. L.A. Steen, editor, *Heeding the Call for Change: Suggestions for Curricular Action*, MAA, 1992.
10. Waterloo Maple, http://www.maplesoft.com.