**Homework #2**
**Due Monday, July 6**

You may discuss these problems with each other, but when it comes time to write up the solutions that you are submitting, you should do this alone.

1. Turn in your Twelve Fold Way notes for me to look at. If you don't want me to mark on them, then submit a copy.

2. Read Sections 1.1–1.6, 3.1–3.3, 12.1. Much of this relates to what we have been doing so far.

3. Prove by induction on $n \geq 0$ that $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$. You may use the fact that $\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$ for $n \geq 1$, $0 \leq k \leq n$.

4. Exercise 3.1.4, page 64.

5. We can turn any composition into a subset by taking "partial sums." For example, the composition $13 = 1+2+1+5+4$ is turned into the subset $\{1, 1+2, 1+2+1, 1+2+1+5, 1+2+1+5+4\} = \{1, 3, 4, 9, 13\}$. Use this to find another proof that the number of compositions of a positive integer $m$ is $2^{m-1}$.

6. Use the Pigeonhole Principle to prove that no matter how a set $S$ of 10 positive integers smaller than 100 is chosen there will always be two different subsets of $S$ that have the same sum.

7. Let $f(m)$ be the number of ways of writing the positive integer $m$ as a composition of 1's, 2's, and 3's.

   (a) Find a recursive formula for $f(m)$, including the base case(s).

   (b) Use this recursive formula to calculate $f(1), \ldots, f(9)$.

   (c) Find a generating function for $f(m)$, so that $f(m)$ is the coefficient of $x^m$ in its series expansion.

   (d) Use this generating function and www.wolframalpha.com to calculate $f(1), \ldots, f(9)$.