MA 515 HOMEWORK #6
Due Wednesday, October 29

ANNOUNCEMENT: There will be no class on Monday, October 27. Instead, there will be an extra class on Wednesday, October 29, 8:00 am, in CB219.

1. Page 149, #4.

2. Page 149, #5.

3. Suppose you have a project that consists of a set $\{1, \ldots, n\}$ of tasks to perform. Associated with each task i is a completion time a_i and a set S_i of tasks that must first all be completed before task i is begun. Assume that task 1 is an artificial "starting task" with $a_1 = 0$, and that task n is an artificial "ending task" with $a_n = 0$. The problem is to complete the entire project in the shortest possible time. Let t_i be a variable that represents the starting time of task i. Formulate this problem as a linear program, and then show that it can be be expressed and solved as a dual of a minimum cost dipath problem. In particular, comment on why there will be no difficulty in solving the minimum cost dipath problem by the simplex method.

4. An $m \times n$ matrix A is *totally unimodular* (TU) if every subdeterminant of A is either 1, -1, or 0. That is to say, for every selection of k rows and k columns (not necessarily adjacent), the $k \times k$ submatrix determined by these rows and columns has determinant either 1, -1, or 0.

   (a) Prove that a matrix A is totally unimodular if and only if any one of the matrices A^T, $-A$, $[A, A]$, $[A, -A]$, $[A, I]$ is totally unimodular.

   (b) Given a $(0, \pm 1)$ matrix A. Prove that *if* both of the following conditions are satisfied, *then* A is totally unimodular.

      i. Each column contains at most two nonzero elements.

      ii. The rows of A can be partitioned into two sets A_1 and A_2 such that two nonzero entries in a column are in the same set of rows if they have different signs and in different sets of rows if they have the same sign.

   Suggestion: First take the case when the submatrix has a column consisting entirely of 0's. Then take the case when every column of the submatrix has exactly two nonzero entries. Then take the case when there exists a column containing exactly one nonzero entry.

   (c) Prove that the converse of the statement in (4b) is false.

   (d) Prove that the node-arc incidence matrix of every directed graph is totally unimodular.