MA515 Homework #4
Due Wednesday, September 28

Let us assume we have an LP in the form

$$\begin{aligned} \max z &= c^T x \\ \text{s.t. } Ax &= b \\ x &\geq O \end{aligned}$$

where the matrix A has full row rank as a result of inserting slack variables. We can represent the data in the form of a tableau T. For example, here is the tableau for the GGMC problem:

x_1	x_2	x_3	x_4	x_5	$-z$	
1	2	1	0	0	0	120
1	1	0	1	0	0	70
2	1	0	0	1	0	100
5	4	0	0	0	1	0

.

Each row represents an equation. For example, the first row represents the equation $x_1 + 2x_2 + x_3 = 120$ and the last row represents the equation $5x_1 + 4x_2 - z = 0$ (which is equivalent to $z = 5x_1 + 4x_2$.) Note the identity matrix associated with the columns for the slack variables and the column $-z$.

Now suppose we are interested in focusing our attention on a different basis for the column space of A, say, $B = \{1, 2, 5\}$. We can perform row operations on the tableau T to result in a tableau T' with an identity matrix in the columns associated with the new basis (and the column labeled by $-z$):

x_1	x_2	x_3	x_4	x_5	$-z$	
1	0	−1	2	0	0	20
0	1	1	−1	0	0	50
0	0	1	−3	1	0	10
0	0	1	−6	0	1	−300

The rows of T' represent a set of four equations equivalent to the original four equations of T.

1. How can you easily read off the associated basic solution $\overline{x}$ from T'? Why does this work in general?

2. How can you easily read off the associated basic directions from T'? Why does this work in general?

3. How can you easily read off the costs of the associated basic directions from T'? Why does this work in general?

4. When contemplating a pivot, how can we determine the entering variable from T'? Why does this work in general?

5. When contemplating a pivot, how can we determine whether the LP has unbounded objective function value from T'? Why does this work in general?

6. When contemplating a pivot, how can we perform the ratio test using the data in T'? Why does this work in general?

7. How can you easily read off the vector $\overline{y}$ from T'? Why does this work in general?