MA515 HOMEWORK #8
Due Wednesday, November 7

1. In a communications network, represented by a digraph, the probability that the edge from i to j is operative is p_{ij}. The probability that all the edges in any given dipath are operative is the product of the edge probabilities. How can one solve the problem of finding a most reliable dipath from one designated vertex to all of the others? Justify your answer.

2. Suppose a digraph is given with nonnegative capacities c_{ij} assigned to the edges. The capacity of a dipath is defined to be the minimum of the capacities of its edges. How can one solve the problem of finding maximum capacity dipaths between every pair of vertices? Justify your answer.

3. Construct an example of a digraph G, with specified vertices v and w, and specified edge weights $c(e)$, $e \in E(G)$, some of which are negative, so that when you carry out Dijkstra's algorithm on this graph the algorithm fails to give the correct weight of a minimum weight v-w dipath.

4. A certain project consists of a set of tasks to be performed. Denote the set of tasks by $\{1, \ldots, n\}$. Each task j requires a certain amount of time a_j (assumed to be nonnegative). For each task j there is a certain subset of tasks $S_j \subseteq \{1, \ldots, n\} \setminus \{j\}$ that must be completed some time before task j is begun. For simplicity, assume that task 1 must be completed before all other tasks are begun, and that all other tasks must be completed before task n is begun. (I.e., task 1 is the initial task and task n is the final task.)

   (a) Using unrestricted variables t_j to represent the time that task j is begun, and the objective function $t_n - t_1$, write a linear program to solve the problem of determining the minimum amount of time required to complete the entire project.

   (b) Prove that the dual linear program is a max weight dipath problem in a certain digraph. (If it isn't, try another formulation of the primal problem.)

   (c) Prove that the shortest completion time equals the weight of a maximum weight dipath between two certain vertices in the digraph.

5. On your own: Be sure to practice carrying out the various minimum weight dipath algorithms on concrete examples.