MA515 HOMEWORK #6
Due Monday, November 9

1. Use Hall's Theorem to prove the following theorem: If G is a bipartite graph, and every vertex has the same degree $k > 1$, then G has a perfect matching—one meeting every vertex. Suggestion: First prove that $|V_1| = |V_2|$. Then prove that $|N(W)| \geq |W|$ for every $W \subseteq V_1$.

2. A *vertex packing* in a graph is a set of vertices, no two of which are joined by an edge. A *covering of vertices by edges* in a graph is a set of edges such that every vertex is an endpoint of at least one of these edges. Prove the following, using the techniques similar to those that we used for the proof of König's Theorem.

   Theorem: If G is a bipartite graph such that every vertex is incident to at least one edge, then the size of a maximum cardinality vertex packing equals the size of a minimum cardinality covering of vertices by edges.

3. Exercise (Bellman-Ford Algorithm), page 76.

4. Problem (Recovering the dipaths with the Bellman-Ford Algorithm), page 77.