MA/STA515 Homework #6
Due Wednesday, October 27

1. Problem 5C. Suggestion: Consider an associated bipartite graph in which edges correspond to nonzero entries of the given matrix.

2. Problem 5F.

3. Define a subset I of vertices of a graph to be *independent* if no two vertices in I are joined by an edge. Define a subset K of edges of a graph to be a *covering of vertices by edges* if every vertex is an endpoint of at least one of the edges of K.

   (a) Let G be any graph. Prove that the size of any independent set of vertices is less than or equal to the size of any covering of vertices by edges.

   (b) Let G be a bipartite graph with no vertices of degree zero. Prove that the size of a maximum cardinality independent set of vertices equals the size of a minimum covering of vertices by edges. Suggestion: Use the theorem that the size of a maximum cardinality matching equals the size of a minimum covering of edges by vertices. Relate independent sets of vertices to coverings of edges by vertices, and coverings of vertices by edges to matchings.

   (c) Give an example that to show that the above theorem is not true for arbitrary graphs.

4. Just a comment to insert in your notes: Problem 3A in the book needs to be reworded to state "...and which is minimal (with respect to the number of vertices) subject to these properties." The intent is that H be a simple graph with all degrees $\leq d$ that cannot be d-colored, and for which all subgraphs of H with fewer vertices can be d-colored. This explains some of the confusion some of you had in attempting to solve this problem.