Assignment #5

1. Note the following change: We will continue to meet regularly in CB313.

2. Read Chapters 1 and 2 of *Beyond the Third Dimension*. There are a fair number of mathematical concepts introduced and discussed, so it will take some time. Make careful note of items and details that you don't understand yet.

3. Write up to turn in. Let's work a bit on making predictions based on analogy.

   (a) A point is just that—a single 0-dimensional point. A line segment has two 0-dimensional endpoints (vertices) and one 1-dimensional edge (itself). A square has four 0-dimensional vertices, four 1-dimensional edges, and one 2-dimensional square (itself). A cube has eight 0-dimensional vertices, twelve 1-dimensional edges, six 2-dimensional squares, and one 3-dimensional cube (itself). Think about how each figure is generated from the previous one by making a copy of it and then connecting up corresponding vertices, edges, etc. Explain why this suggests that the number of k-dimensional elements of an object equals twice the number of k-dimensional elements in the preceding object plus the number of $(k-1)$-dimensional elements in the preceding object. Based on this, make predictions for:

      i. The number of elements of dimensions 0, 1, 2, 3, and 4 of a four-dimensional cube.

      ii. The number of elements of dimensions 0, 1, 2, 3, 4, and 5 of a five-dimensional cube.

      iii. The number of elements of dimensions 0, 1, 2, 3, 4, 5, and 6 of a six-dimensional cube.

   (b) Try to figure out what this has to do with expanding the polynomial $(2+x)^d$ for $d = 0, 1, 2, 3, 4, 5, 6, \ldots$. For example, what happens when you calculate $(2+x)^3$ by first calculating $(2+x)^2$ and then multiplying the result by $(2+x)$?

4. Write up to turn in. You can center a square in the plane at the origin so that its vertices have coordinates $(\pm 1, \pm 1)$. Similarly you can center a cube in 3-dimensional space so that its vertices have coordinates $(\pm 1, \pm 1, \pm 1)$.

   (a) What does analogy suggest you can choose for the coordinates of the vertices of a 4-dimensional cube?

   (b) How can you tell, just by looking at the coordinates, which pairs of vertices in the 3-cube or 4-cube are joined by edges?

(c) How can you tell, just by looking at the coordinates, which quadruples of vertices in the 3-cube or 4 cube form a square?

(d) How can you tell, just by looking at the coordinates, which sets of eight vertices of the 4-cube form a cube?