## Exam 3

Form A

Name: $\qquad$ Section and/or TA:
Do not remove this answer page - you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 11 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. The wise student will show work for the multiple choice problems as well.

## Multiple Choice Questions

2 A
(B)
(C)
(D)
(E)
7 A
(B)
(C)
(D)
(E)
3
(A) (B) (D) E
8 (A)
(B) (C)
(D)
(E)
4
(A)
(B)
(C)
(D)
(E)
9 (A)
(B)
(C) (D)
(E)
5
(A)
(B)
(C) D
(E)
10 (A)
(B) (C)
(D) (E)
6
(A)
(B)
(C)
(D)
(E)
11
(C)
(D) (E)

## SCORE

| 1 | Multiple <br> Choice | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | Total <br> Score |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 50 | 6 | 6 | 6 | 3 | 9 | 6 | 4 | 3 | 3 | 3 | 100 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

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## Multiple Choice Questions

1. Take a deep breath and write "I got this ...".
2. Expand the following logarithmic expression

$$
\ln \left(\frac{(x+8)(x-4)}{(x-2)^{2}}\right)^{3 / 5}, \quad x>4
$$

A. $\frac{3}{5} \ln (x+8)+\frac{3}{5} \ln (x-4)-\frac{6}{5} \ln (x-2)$
B. $3 \ln (x+8)-5 \ln (x-4)-\frac{6}{5} \ln (x-2)$
C. $\frac{3}{5} \ln \left(x^{2}+12 x-32\right)-\frac{6}{5} \ln (x-2)$
D. $\ln (x+8)+\ln (x-4)+\ln 36 \ln (x-2)-\ln 5$
E. $\frac{3}{5} \ln (x+8)+\frac{3}{5} \ln (x-4)-\frac{3}{5} \ln (x-2)$
3. Express as a single logarithm:

$$
3 \log _{a} x-\frac{5}{3} \log _{a} y+\frac{1}{6} \log _{a} w-5 \log _{a} z
$$

A. $\log _{a} \frac{x^{3} z^{5}}{w^{1 / 6} y^{5 / 3}}$
B. $\log _{a} \frac{x^{3} w^{1 / 6}}{y^{5 / 3} z^{5}}$
C. $\log _{a} \frac{x^{3} y^{5 / 3}}{w^{1 / 6} z^{5}}$
D. $\log _{a} \frac{x^{3}}{w^{1 / 6} y^{5 / 3} z^{5}}$
E. $\log _{a}\left(3 x-\frac{5}{3} y+\frac{1}{6} w-5 z\right)$
4. Solve the following equation for $x$.

$$
\log (x+9)=1-\log x
$$

A. $\{-10,1\}$
B. $\{1\}$
C. $\{-1\}$
D. $\{-1,10\}$
E. No solution
5. Describe how to transform the graph of the basic function $g(x)=\log x$ into the graph of the function $f(x)=9 \log (4-x)$.
A. Reflect across the $y$-axis, translate 4 units to the right, and vertically stretch by a factor of 9 .
B. Reflect across the $y$-axis, translate 4 units to the left, and vertically stretch by a factor of 9 .
C. Reflect across the $x$-axis, translate 4 units to the left, and vertically stretch by a factor of 9 .
D. Reflect across the $x$-axis, translate 4 units to the right, and vertically stretch by a factor of 9 .
E. Reflect across the $y$-axis, translate 4 units to the right, and vertically compress by a factor of 9 .
6. Convert $162^{\circ}$ to radians.
A. $\frac{9 \pi}{10}$
B. $\pi$
C. $\frac{4 \pi}{5}$
D. $\frac{9 \pi}{5}$
E. $\frac{4 \pi}{10}$
7. Use basic trigonometric identities to simplify

$$
\frac{\cos ^{2} x}{\sin ^{2} x}+\cos x \sec x
$$

A. $\csc x$
B. $\sec ^{2} x$
C. $\tan x$
D. $\cot ^{2} x$
E. $\csc ^{2} x$
8. Solve the following equation for $x$ :

$$
9^{2 x} \cdot 27^{3-x}=\frac{1}{9}
$$

A. $\{-10\}$
B. $\{-11\}$
C. $\left\{\frac{9+\sqrt{87}}{6}, \frac{9-\sqrt{87}}{6}\right\}$
D. $\{-8\}$
E. $\{10\}$
9. Find $\log _{3} 13$.
A. 2.334718
B. 2.801661
C. 2.334318
D. 0.428317
E. 3.344718
10. Find the exponential function whose graph is given

A. $y=-4^{x}$
B. $y=4^{x+1}$
C. $y=4^{-x}$
D. $y=x^{4}$
E. $y=4^{x}$
11. A 50-gallon barrel is filled completely with pure water. Salt water with a concentration of $0.5 \mathrm{lb} / \mathrm{gal}$ is then pumped into the barrel, and the resulting mixture overflows at the same rate. The amount of salt in the barrel at time $t$ is given by

$$
Q(t)=25\left(1-e^{-0.08 t}\right)
$$

where $t$ is measured in minutes and $Q(t)$ is measured in pounds. How much salt is in the barrel after 10 min ?
A. 15.9 pounds
B. 15.8 pounds
C. 14.8 pounds
D. 13.8 pounds
E. 13.7 pounds

## Free Response Questions

12. Suppose you're driving your car on a cold winter day ( $18^{\circ} \mathrm{F}$ outside) and the engine overheats (at about $220^{\circ} \mathrm{F}$ ). When you park, the engine begins to cool down. The temperature $H$ of the engine $t$ minutes after you park satisfies the equation

$$
\ln \left(\frac{H-18}{202}\right)=-0.13 t
$$

(a) Solve the equation for $H$.
(b) Use part (a) to find the temperature of the engine after $25 \mathrm{~min}(t=25)$. Round your answer to two decimal places.
13. Given that the terminal point for angle $\theta$ is $\left(\frac{5}{8},-\frac{7}{8}\right)$, find
(a) $\sin \theta$
(b) $\cos \theta$
(c) $\tan \theta$
(d) $\cot \theta$
(e) $\sec \theta$
(f) $\csc \theta$
14. Given that $\sec \theta=7$ and $\sin \theta<0$, find the values of the other trigonometric functions of $\theta$.
(a) $\sin \theta$
(b) $\cos \theta$
(c) $\tan \theta$
(d) $\cot \theta$
(e) $\sec \theta$
(f) $\csc \theta$
15. Find the domain of the function $f(x)=\ln \left(x^{2}-16\right)$.
16. Vilfredo Pareto (1848-1923) observed that most of the wealth of a country is owned by a few members of the population. Pareto's Principle is

$$
\log P=\log c-k \log W
$$

where $W$ is the wealth level (how much money a person has) and $P$ is the number of people in the population having that much money.
(a) Solve the equation for $P$.
(b) Assume $k=2.5, c=7,000$, and $W$ is measured in millions of dollars. Use part (a) to find the number of people who have $\$ 2$ million or more. Round the answer to the nearest integer.
(c) How many people have $\$ 11$ million or more? Again round the answer to the nearest integer.
17. The elk population in a certain region is given by the function $E(t)=1050+150 \sin \left(\frac{4 t}{5}\right)$ where the time $t$ is measured in years.
(a) What is the largest number of elk present in the region at any time?
(b) How much time elapses between occurrences of the largest and smallest elk population?
18. Find the amplitude, period and phase shift of $-2 \sin \left(2 x-\frac{\pi}{4}\right)$. Sketch a graph of the function on the supplied axes.

19. Simplify the expression $\frac{\sin ^{2} x-\cos ^{2} x}{1-\tan ^{2} x}$.
20. Determine the equation of the function that is graphed below:

21. Use the properties of logarithms to expand the expression

$$
\log \sqrt[6]{x \sqrt[6]{y \sqrt[6]{z}}}
$$

in a form with no logarithm of a product, quotient, or power.

