MA 110 Algebra and Trigonometry for Calculus — Spring 2017
Exam 4 — Monday, 1 May 2017

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has twelve multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

EXAMPLE	A	B	C	D	E
Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
13		10
14		10
15		10
16		10
17		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

The following identities and formulae may be useful. All assume our standard naming convention for triangles where the side of length a is opposite the angle of measure A, the side of length b is opposite the angle of measure B and the side of length c is opposite the angle of measure C.

$\sin(x) = \cos(\frac{\pi}{2} - x)$ $\quad$ $\sec(x) = \csc(\frac{\pi}{2} - x)$ $\quad$ $\cot(x) = \tan(\frac{\pi}{2} - x)$

$\tan(x) = \cot(\frac{\pi}{2} - x)$ $\quad$ $\cos(x) = \sin(\frac{\pi}{2} - x)$ $\quad$ $\csc(x) = \sec(\frac{\pi}{2} - x)$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$
$$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

$$\sin(2x) = 2\sin(x)\cos(x) \qquad \tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) \qquad \cos(2x) = 1 - 2\sin^2(x) \qquad \cos(2x) = 2\cos^2(x) - 1$$

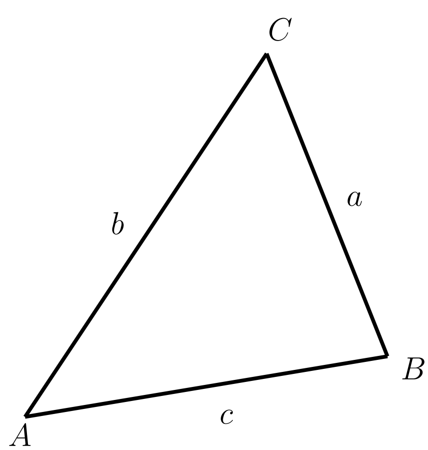
$$\sin(\tfrac{x}{2}) = \pm\sqrt{\frac{1-\cos(x)}{2}} \qquad \cos(\tfrac{x}{2}) = \pm\sqrt{\frac{1+\cos(x)}{2}} \qquad \tan(\tfrac{x}{2}) = \pm\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$$

Law of sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C).$$



Record the correct answer to the following problems on the front page of this exam.

1. Solve.

$$\frac{x}{5} = \frac{1}{x-6}$$

(a) $x = -5$ and $x = 1$
(b) No real solutions
(c) $x = -1$ and $x = 5$
(d) $x = 3 \pm \sqrt{14}$
(e) $x = \sqrt{10}$

2. Find $g^{-1}(-7)$

$$g(x) = \frac{x-2}{x+3}.$$

(a) $\frac{9}{4}$
(b) $-\frac{1}{2}$
(c) $-\frac{19}{8}$
(d) $\frac{3}{2}$
(e) $-\frac{23}{8}$

3. Given the following functions, find the remainder when $f(x)$ is divided by $g(x)$.

$$f(x) = x^4 - 3x^2 + x - 5 \qquad \text{and} \qquad g(x) = x + 2$$

(a) -1
(b) 1
(c) -27
(d) -3
(e) 27

Record the correct answer to the following problems on the front page of this exam.

4. Solve.

$$\log_7(8-x) - \log_7(x+1) = \log_7(x)$$

   (a) $x = -4$ and $x = 2$
   (b) $x = -2\sqrt{2}$ and $x = 2\sqrt{2}$
   (c) $x = 2$
   (d) $x = 2\sqrt{2}$
   (e) $x = 6$

5. Assume that $\cos(\theta) = 5/13$ and $-\pi/2 \leq \theta \leq 0$. Find $\sin(\theta)$. (The angle θ is measured in radians.)

   (a) $\sin(\theta) = 5/13$
   (b) $\sin(\theta) = -12/13$
   (c) $\sin(\theta) = 12/13$
   (d) $\sin(\theta) = -5/13$
   (e) $\sin(\theta) = -12/5$

6. Find the exact value.

$$\sin\left(\frac{7\pi}{20}\right)\cos\left(\frac{3\pi}{5}\right) - \cos\left(\frac{7\pi}{20}\right)\sin\left(\frac{3\pi}{5}\right)$$

   (a) $\frac{\sqrt{3}}{2}$
   (b) $\frac{1}{2}$
   (c) $-\frac{\sqrt{3}}{2}$
   (d) $-\frac{1}{2}$
   (e) $-\frac{\sqrt{2}}{2}$

Record the correct answer to the following problems on the front page of this exam.

7. Find the exact value of the following expression:

$$\cos\left(\frac{7\pi}{8}\right)$$

(a) $\frac{\sqrt{2+\sqrt{2}}}{2}$

(b) $\frac{\sqrt{2+\sqrt{3}}}{2}$

(c) $-\frac{\sqrt{2-\sqrt{2}}}{2}$

(d) $-\frac{\sqrt{2+\sqrt{2}}}{2}$

(e) $\frac{\sqrt{2-\sqrt{2}}}{2}$.

8. Find the **exact** value. $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

(a) Undefined

(b) $\frac{2\pi}{3}$

(c) $\frac{\sqrt{3}}{2}$

(d) 1.0472

(e) $\frac{\pi}{3}$

9. Write the following as an algebraic expression in v.

$$\tan\left(\cos^{-1}(v)\right)$$

(a) $\dfrac{\sqrt{1-v^2}}{v}$ $\quad -1 \leq v \leq 1$

(b) $\dfrac{v}{\sqrt{1+v^2}}$ $\quad -1 \leq v \leq 1$

(c) $\dfrac{v}{\sqrt{1-v^2}}$ $\quad -1 \leq v \leq 1$

(d) $\dfrac{1}{\sqrt{1-v^2}}$ $\quad -1 \leq v \leq 1$

(e) $\dfrac{1}{\sqrt{1+v^2}}$ $\quad -1 \leq v \leq 1$

Record the correct answer to the following problems on the front page of this exam.

10. Use factoring, the quadratic formula, or identities to solve the equation. Find all solutions in the interval $[0, 2\pi)$.

$$\cos(2x) - \sin(x) = 1$$

(a) $x = 0, \pi$

(b) $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

(c) $x = 0, -\pi$

(d) $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

(e) $x = \frac{7\pi}{6}, -\frac{7\pi}{6}$

11. Solve ΔABC under the given conditions.

$$A = 140^\circ, \qquad b = 11, \qquad c = 19$$

(a) $a = 28.3, \qquad B = 14.5^\circ, \qquad C = 25.5^\circ$

(b) $a = 14.6, \qquad B = 16.1^\circ, \qquad C = 23.9^\circ$

(c) $a = 28.3, \qquad B = 11.4^\circ, \qquad C = 28.6^\circ$

(d) $a = 29.5, \qquad B = 19.7^\circ, \qquad C = 20.3^\circ$

(e) $a = 14.6, \qquad B = 12.2^\circ, \qquad C = 27.8^\circ$

12. Jack and Jill are standing 20 feet apart on opposite sides of a tree. His angle of elevation to the top of the tree is 45°, while Jill is closer and measures her angle of elevation at 75°. How tall is the tree?

(a) 22.3 feet

(b) 20.0 feet

(c) 19.0 feet

(d) 15.8 feet

(e) 16.3 feet

Free Response Questions: Show your work!

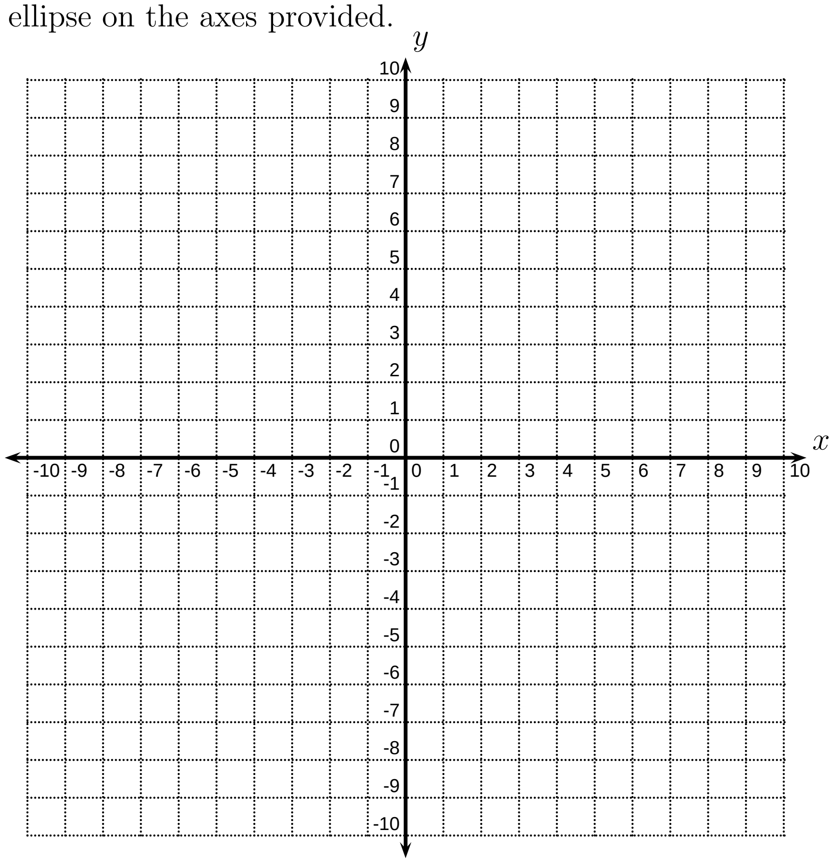
13. Consider the ellipse given by the equation

$$20x^2 + 9y^2 - 40x + 54y = 79.$$

(a) Use "completing the square" to put the equation in standard form. Then give the **EXACT** coordinates of the center and vertices of the ellipse.

Center __________ Vertices __________ __________

(b) Sketch the ellipse on the axes provided.

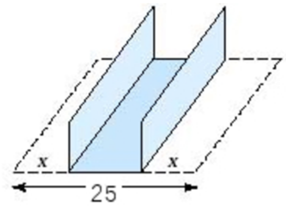


Free Response Questions: Show your work!

14. A gutter is to be made by bending up the sides of a 25-inch-wide piece of aluminum.

Let y be the width of the gutter.

Let x be the depth of the gutter.



(a) Write an equation for the width y in terms of x.

(b) Write an equation for the cross-sectional area of the gutter in terms of x.

(c) Use any method to determine what depth the gutter should be to have the maximum possible cross-sectional area.

(d) What will the width of the maximum area gutter be?

(e) What is the maximum cross-sectional area?

Free Response Questions: Show your work!

15. The antibiotic clarithromycin is eliminated from the body according to the formula $A(t) = 500e^{-0.1386t}$, where A is the amount remaining in the body (in milligrams) t hours after the drug reaches peak concentration.

    (a) How much antibiotic is remaining after 2 hours and 40 minutes? Round your answer to the nearest tenth of a milligram.

    (b) How much time will pass before the amount of drug in the body is reduced to 100 milligrams? Round your answer to the nearest tenth of an hour.

    (c) Find the inverse of $A(t)$ and explain what the inverse function models.

Free Response Questions: Show your work!

16. Use factoring, the quadratic formula, or identities to find ALL solutions of the equations below. You must use algebraic methods and show your work. ONLY **exact answers** in terms of radians will receive full credit!

    (a)

    $$\sqrt{3}\tan(x) + 1 = 0$$

    (b)

    $$\sin\left(\frac{x}{2}\right) = 1 - \cos(x)$$

Free Response Questions: Show your work!

17. A forest fire is spotted from two different fire towers. The triangle determined by the two towers and the fire has angles of $20°$ and $29°$ at the tower vertices, T1 and T2, respectively. The towers are 2,775 meters apart.

    (a) Summarize the given information in a sketch. Your sketch should clearly label the triangle you use to answer the following parts.

    (b) How far is T1 from the fire?

    (c) How far is T2 from the fire?

    (d) From which tower should firefighters be dispatched? Why?