

KEY

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has twelve multiple choice questions (5 points each), five true/false questions (2 points each) and three free response questions (10 points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has scientific or graphing capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the true/false choice problems:

1. You must give your *final answers* in the *true/false choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *true/false choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*).
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

EXAMPLE	A	B	C	D	E
Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

True/False Choice Answers

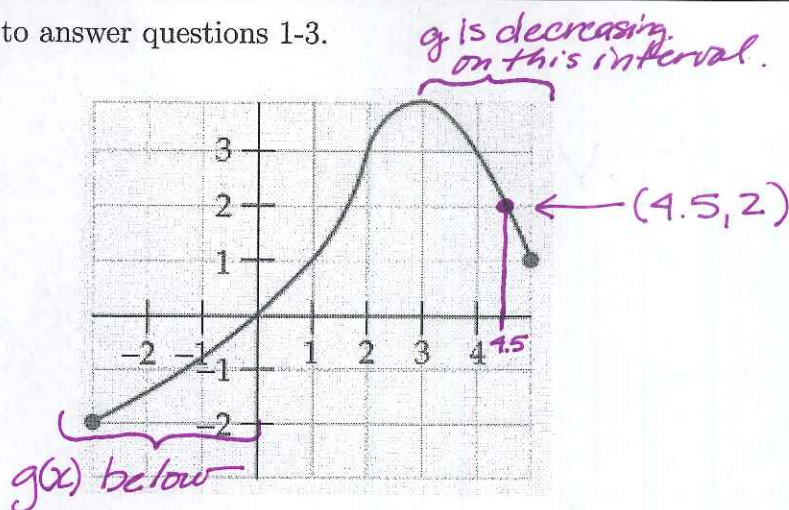
Question		
13	T	F
14	T	F
15	T	F
16	T	F
17	T	F

Exam Scores

Question	Score	Total
MC		60
TF		10
18		10
19		10
20		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

Use the graph of g below to answer questions 1-3.



1. If $t = 2$, then $g(t + 2.5) =$ _____.

- (a) 1
- (b) 2
- (c) 3
- (d) 4.5
- (e) 5

$$\begin{aligned} t = 2 &\Rightarrow g(t + 2.5) = g(2 + 2.5) \\ &= g(4.5) \\ &= 2 \end{aligned}$$

2. For what values of x is $g(x) < 0$?

- (a) $[-3, 0)$
- (b) $[-2, 0)$
- (c) $(0, 3]$
- (d) $(0, 4]$
- (e) $(0, 5]$

$$\begin{aligned} g(x) < 0 &\Rightarrow g(x) \text{ is } \underline{\text{below}} \text{ } x\text{-axis} \\ &\text{This happens for } x\text{-values} \\ &\underline{[-3, 0)}. \end{aligned}$$

3. Determine the approximate interval where g is decreasing.

- (a) $[-2, 3]$
- (b) $(-2, 4)$
- (c) $[0, 5]$
- (d) $(3, 5)$
- (e) $(2, 4)$

$$\begin{aligned} g \text{ is decreasing} &\Rightarrow g \text{ is falling from left to right} \\ &\text{This happens for } x\text{-values} \\ &\underline{(3, 5]}. \end{aligned}$$

Record the correct answer to the following problems on the front page of this exam.

4. The rule of the function g is given by the graph below. Determine the range.

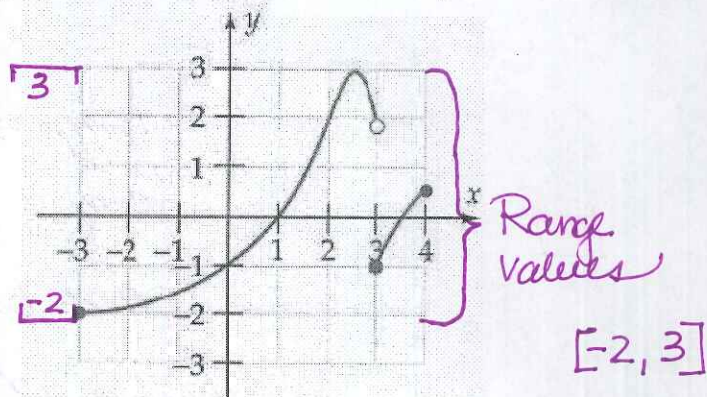
(a) $[-3, 3]$

(b) $[-2, 3]$

(c) None of the other choices.

(d) $[-3, 4]$

(e) $[-2, 4]$



5. Jack and Jill are salespersons in the suit department of a clothing store. Jack is paid \$200 per week plus \$5 for each suit he sells, whereas Jill is paid \$10 for every suit she sells. If $f(x)$ represents Jack's weekly income and $g(x)$ represents Jill's weekly income from selling x suits, then determine the rules of the functions f and g .

(a) $f(x) = 200x + 5$; $g(x) = 10$

(b) None of the other choices.

(c) $f(x) = 200x + 5$; $g(x) = 10x$

(d) $f(x) = 5x + 200$; $g(x) = 10x$

(e) $f(x) = 5x + 200$; $g(x) = 10$

$x = \#$ of suits sold

Jack = $200 + \$5(\# \text{ of suits})$

$f(x) = 200 + 5x$

Jill = $\$10(\# \text{ of suits})$

$g(x) = 10x$

6. If $g(t) = t^2 - t$ and $f(x) = 1 + x$, then compute $g(f(2) + 3)$.

(a) 3

(b) 9

(c) 6

(d) 30

(e) 20

Record the correct answer to the following problems on the front page of this exam.

7. In a laboratory culture, the number $N(d)$ of bacteria (in thousands) at temperature d degrees Celsius is given by the function $N(d) = -\frac{90}{d+1} + 20$. The temperature $d(t)$ at time t hours is given by the function $d(t) = 2t + 4$. How many bacteria are in the culture after 4 hours?

(a) $\approx 2,000$

(b) $\approx 8,000$

(c) $\approx 13,077$

(d) None of the other choices.

(e) $\approx 16,400$

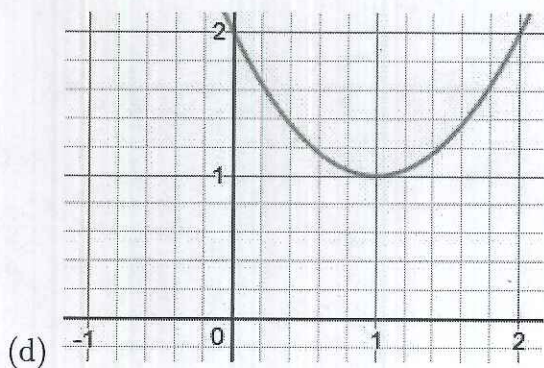
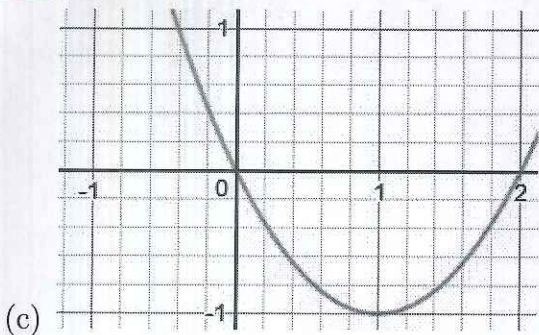
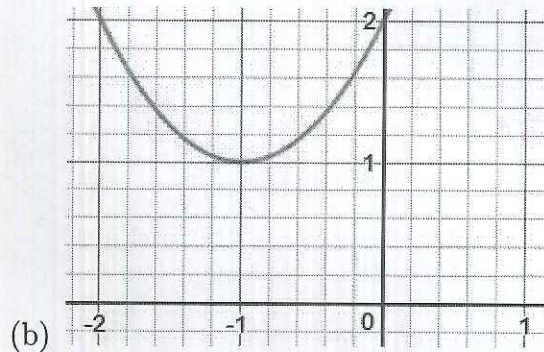
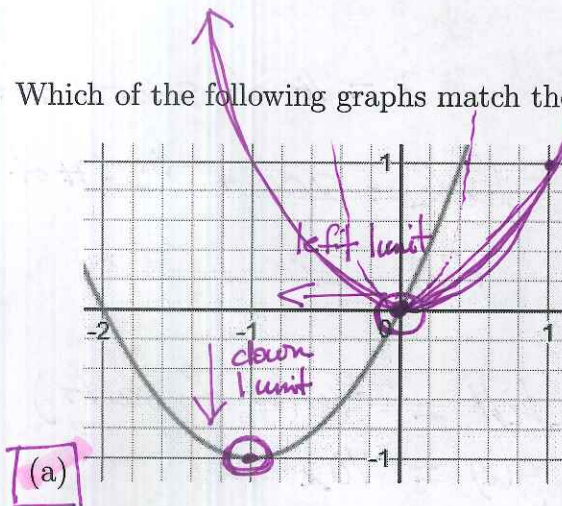
temp $\Rightarrow d(t) = 2t + 4$ where $t = \text{time}$
 temp @ 4 hrs. $\Rightarrow d(4) = 2(4) + 4 = 12^\circ\text{C}$

of bact. $\Rightarrow N(d) = -\frac{90}{d+1} + 20$ where $d = \text{temp}$
 bact @ $12^\circ\text{C} \Rightarrow N(12) = -\frac{90}{12+1} + 20$

≈ 13.0769 (thousands)

$\approx 13,077$ bacteria

8. Which of the following graphs match the function $f(x) = (x + 1)^2 - 1$?



(e) None of the other choices.

$f(x) = (x+1)^2 - 1$
 ↑ shifts x^2 left 1 unit
 ↑ shifts x^2 down 1 unit

Record the correct answer to the following problems on the front page of this exam.

9. The table below shows the population $f(t)$ of rabbits on Christy's property t years after she received 10 of them as a gift. Determine $3 \cdot f^{-1}(70)$.

t	$f(t)$
0	10
1	23
2	48
3	64
4	70
5	71

(a) None of the other choices.

(b) 12

(c) 4

(d) 210

(e) 280

$$3 \cdot f^{-1}(70)$$

$$3 \cdot 4$$

$$12$$

$$\rightarrow f(4) = 70$$

$$4 = f^{-1}(70)$$

10. If $f(x) = 3x - 2$ and $g(x) = x^2$, then compute $(g \circ f)(3)$.

(a) 9

(b) 25

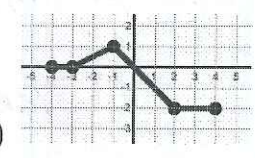
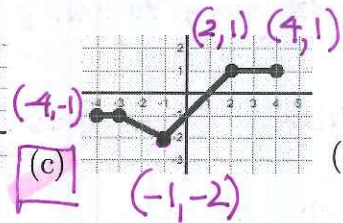
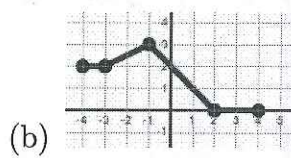
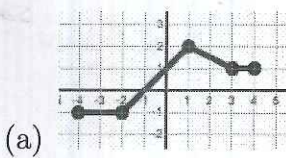
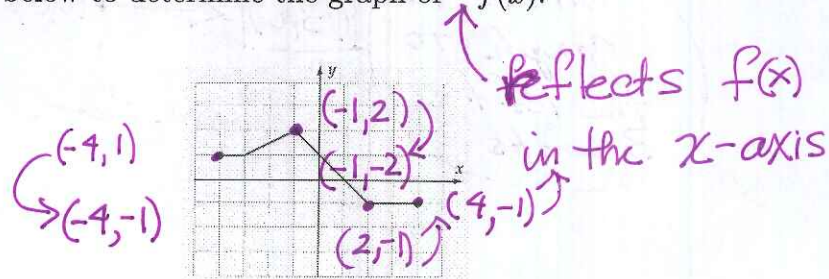
(c) 27

(d) 63

(e) None of the other choices.

$$\begin{aligned} & \cancel{g(f(3))} \\ & g(3(3)-2) \\ & g(7) \end{aligned} \rightarrow (7)^2 = \boxed{49}$$

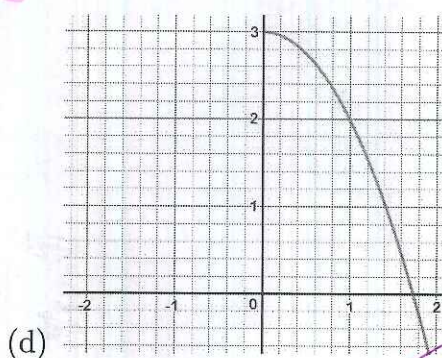
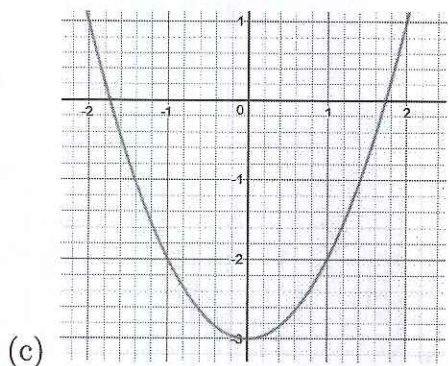
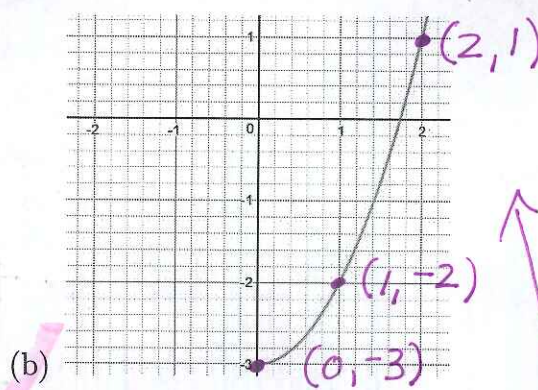
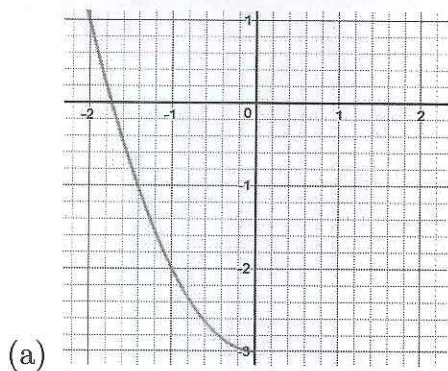
11. Use the graph of $f(x)$ below to determine the graph of $-f(x)$.



(e) None of the other choices.

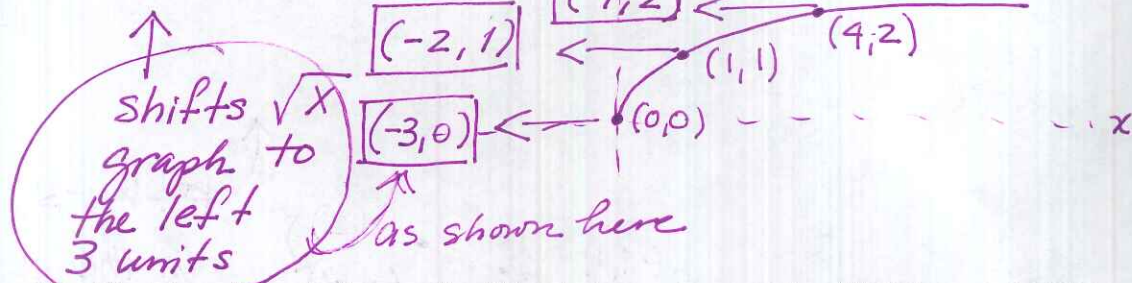
Record the correct answer to the following problems on the front page of this exam.

12. The function $f(x) = \sqrt{x+3}$ has an inverse. Which figure below is the graph of the inverse?



(e) None of the other choices.

$$f(x) = \sqrt{x+3}$$



For questions 13 - 17, determine whether each of the statements are either **TRUE** or **FALSE**.

- T 13. A function may have different inputs with the same output.
Multiple inputs going to the same output is okay, just not multiple outputs for same input.
- T 14. The graph of a function has the property where no vertical line intersects the graph more than once.
Vertical Line Test
- F 15. The graph of $f(x) + 3$ would be the graph of $f(x)$ shifted to the right 3 units.
↑ affects the graph vertically... up 3 units.
- F 16. Every function has an inverse.
Only 1-1 functions have inverses.
- T 17. If a function and its inverse are graphed on the same axes, then they are symmetric about the line $y = x$.
as demonstrated in #12 above.

Free Response Questions: Show your work!

18. For the function $f(x) = x^2 - x$, compute and simplify the difference quotient:

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - (x+h) - [x^2 - x]}{h} \quad +2 \text{ setup}$$

$$= \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h} \quad +2 \text{ expand}$$

$$= \frac{2xh + h^2 - h}{h} \quad +2 \text{ eliminate like terms}$$

$$= \frac{h(2x + h - 1)}{h} \quad +1 \text{ factor}$$

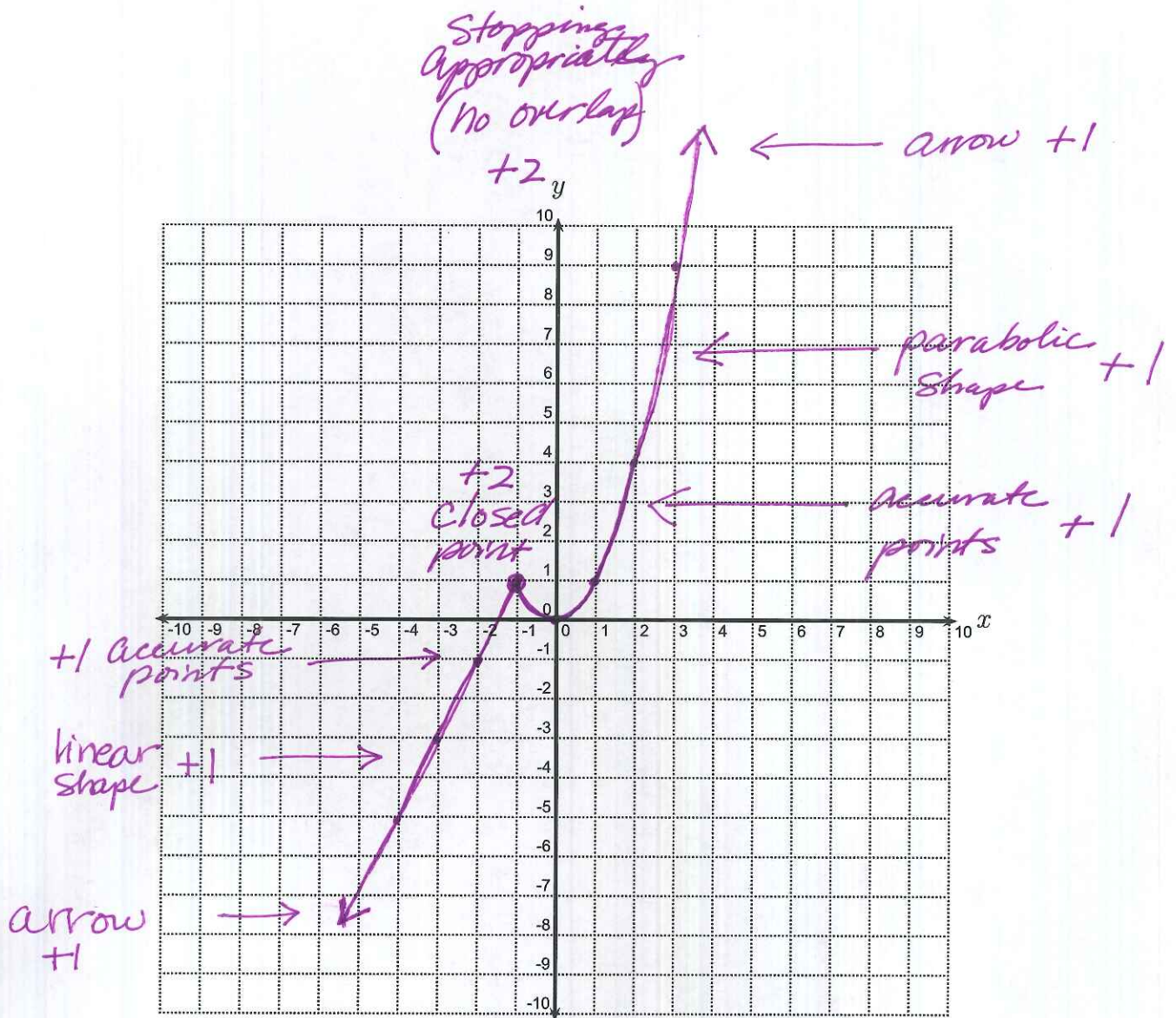
$$= \boxed{2x + h - 1} \quad +1 \text{ divide "h" away}$$

+2 ALL notation (equal signs, minus signs, etc.)

Free Response Questions: Show your work!

19. Sketch the graph of the following function, being sure to indicate which endpoints are included and which ones are excluded.

$$f(x) = \begin{cases} x^2, & x \geq -1 \\ 2x + 3, & x < -1 \end{cases}$$



Free Response Questions: Show your work!

20. Use algebra to compute the inverse of the following one-to-one function:

$$f(x) = \frac{3x}{2-x}$$

$$y = \frac{3x}{2-x}$$

$$\Rightarrow y(2-x) = 3x \quad +1 \text{ multiply by LCD}$$

$$\Rightarrow 2y - xy = 3x \quad +1 \text{ distribute}$$

$$\Rightarrow 2y = 3x + xy \quad +2 \text{ add "xy"}$$

$$\Rightarrow 2y = x(3+y) \quad +2 \text{ factor "x"}$$

$$\Rightarrow \frac{2y}{3+y} = x \quad +1 \text{ divide}$$

$$\Rightarrow y = \frac{2x}{3+x} \quad +1 \text{ switch } x \text{ \& } y$$

$$\Rightarrow f^{-1}(x) = \frac{2x}{3+x} \quad +1 \text{ label } f^{-1}$$

+1 ALL notation