

Name: KEY

Section: _____

Last 4 digits of student ID #: _____

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- You must give your *final answers* in the *multiple choice answer box* on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*).
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

EXAMPLE	A	B	C	D	E
Question					
1	A	<input checked="" type="radio"/>	C	D	E
2	<input checked="" type="radio"/>	B	C	D	E
3	A	B	C	D	<input checked="" type="radio"/>
4	A	B	C	<input checked="" type="radio"/>	E
5	A	<input checked="" type="radio"/>	C	D	E
6	A	B	C	<input checked="" type="radio"/>	E
7	A	B	<input checked="" type="radio"/>	D	E
8	A	B	C	D	<input checked="" type="radio"/>
9	<input checked="" type="radio"/>	B	C	D	E
10	A	B	C	D	<input checked="" type="radio"/>

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Which of the following statements is equivalent to the given algebraic statement?

$$|x - 7| \leq 5$$

- (a) The distance between x and 7 is greater than or equal to 5 units.
(b) The distance between x and 7 is less than or equal to 5 units.
(c) The distance between x and -7 is less than 5 units.
(d) The distance between x and -7 is less than or equal to 5 units.
(e) The distance between x and -7 is greater than or equal to 5 units.

2. Find all the real solutions.

$$9(t - 5) + 4 = 3 - 7(5 - t)$$

- (a) $t = \frac{9}{2}$
(b) $t = 0$ and $t = 5$
(c) No real solutions
(d) $t = \frac{11}{16}$
(e) $t = 5$

$$9t - 45 + 4 = 3 - 35 + 7t$$

$$2t - 41 = -32$$

$$2t = 9$$

$$t = \frac{9}{2}$$

3. Find the exact distance between the two points $(-3, 10)$ and $(-7, -2)$ in the Cartesian plane.

- (a) $\pm 8\sqrt{2}$
(b) 12.65
(c) $\sqrt{194}$
(d) $2\sqrt{41}$
(e) $4\sqrt{10}$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(-3 + 7)^2 + (10 + 2)^2}$$

$$d = \sqrt{4^2 + 12^2}$$

$$d = \sqrt{160} \Rightarrow \sqrt{16} \sqrt{10}$$

$$d = 4\sqrt{10}$$

Record the correct answer to the following problems on the front page of this exam.

4. Solve.

$$\frac{x}{3} = \frac{2}{x-4}$$

- (a) $x = -5$ and $x = 1$
- (b) No real solutions
- (c) $x = -1$ and $x = 5$
- (d) $x = 2 \pm \sqrt{10}$
- (e) $x = \sqrt{10}$

$$x(x-4) = 6$$

$$x^2 - 4x + 4 = 6 + 4$$

$$(x-2)^2 = 10$$

$$x-2 = \pm\sqrt{10}$$

$$x = 2 \pm \sqrt{10}$$

5. Solve.

$$2|4-3x| + 2 > 12.$$

- (a) $(-\infty, -3) \cup (\frac{1}{3}, \infty)$
- (b) $(-\infty, -\frac{1}{3}) \cup (3, \infty)$
- (c) $(-\frac{1}{3}, \infty)$
- (d) $(-\infty, 3)$
- (e) $(-\frac{1}{3}, 3)$

$$2|4-3x| > 10$$

$$|4-3x| > 5$$

$$4-3x > 5$$

$$-3x > 1$$

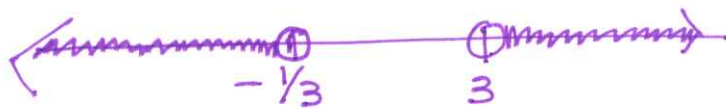
$$x < -\frac{1}{3}$$

$$-(4-3x) > 5$$

$$4-3x < -5$$

$$-3x < -9$$

$$x > 3$$



$$(-\infty, -\frac{1}{3}) \cup (3, \infty)$$

Record the correct answer to the following problems on the front page of this exam.

6. Find a real number k such that the line, $5x - ky + 4 = 0$, has a y -intercept of 3.

(a) $k = -3$

(b) $k = \frac{4}{9}$

(c) $k = -\frac{5}{4}$

(d) $k = \frac{4}{3}$

(e) $k = -\frac{5}{16}$

y -intercepts occur when $x=0$

$$5(0) - k(3) + 4 = 0$$

$$-3k = -4$$

$$k = \frac{4}{3}$$

So the point $(0, 3)$ needs to satisfy the equation.

7. Given the following equation of an ellipse, find its center, vertices and foci.

Center: $(h, k) \Rightarrow (1, -3)$

$$\frac{(x-1)^2}{36} + \frac{(y+3)^2}{20} = 1$$

(Handwritten annotations: h points to $(x-1)$, k points to $(y+3)$, 6^2 points to 36, $\sqrt{20}^2$ points to 20)

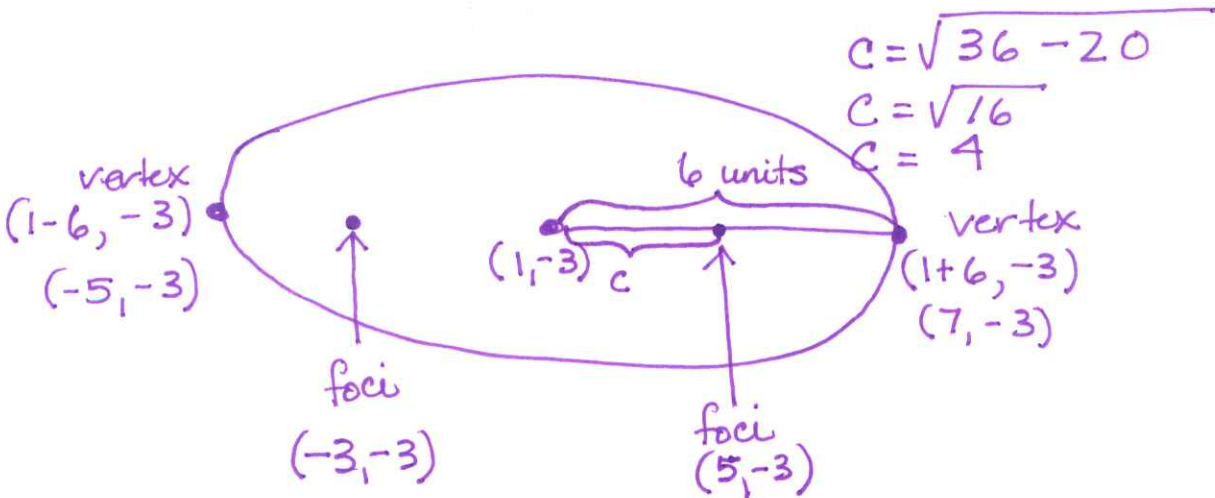
(a) Center $(1, -3)$; Vertices $(1, -3 + \sqrt{20})$ and $(1, -3 - \sqrt{20})$; Foci $(1, 1)$ and $(1, -7)$

(b) Center $(-1, 3)$; Vertices $(-7, 3)$ and $(5, 3)$; Foci $(-3, 3)$ and $(1, 3)$

(c) Center $(1, -3)$; Vertices $(-5, -3)$ and $(7, -3)$; Foci $(-1, -3)$ and $(3, -3)$

(d) Center $(-1, 3)$; Vertices $(1, 3 + \sqrt{20})$ and $(1, 3 - \sqrt{20})$; Foci $(1, -1)$ and $(1, -5)$

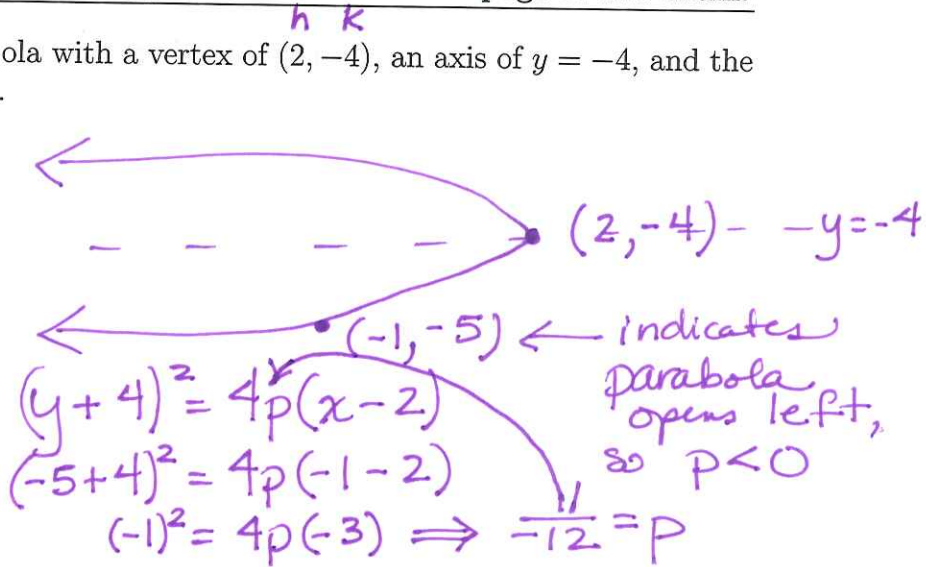
(e) Center $(1, -3)$; Vertices $(-5, -3)$ and $(7, -3)$; Foci $(1, -1)$ and $(1, -5)$



Record the correct answer to the following problems on the front page of this exam.

8. Find the equation of the parabola with a vertex of $(2, -4)$, an axis of $y = -4$, and the point $(-1, -5)$ is on the graph.

- (a) $(y - 2)^2 = \frac{1}{3}(x - 4)$
 (b) $(y + 4)^2 = -\frac{1}{4}(x - 3)$
 (c) $(y + 4)^2 = \frac{1}{3}(x - 2)$
 (d) $(y + 2)^2 = \frac{1}{3}(x - 4)$
 (e) $(y + 4)^2 = -\frac{1}{3}(x - 2)$
- $(y + 4)^2 = 4(-\frac{1}{12})(x - 2)$
 $(y + 4)^2 = -\frac{1}{3}(x - 2)$



9. Consider the equation

$$y^2 - x = -5.$$

Which of the following is correct?

- (a) The equation defines x as a function of y but **not** y as a function of x .
 (b) The equation defines x as a function of y and y as a function of x .
 (c) The equation defines y as a function of x but **not** x as a function of y .
 (d) The equation does **not** define y as a function of x **nor** x as a function of y .
 (e) None of the above options is correct.

$$-x = -y^2 - 5$$

$$x = y^2 + 5 \leftarrow x \text{ is a function of } y$$

$$y^2 = x - 5$$

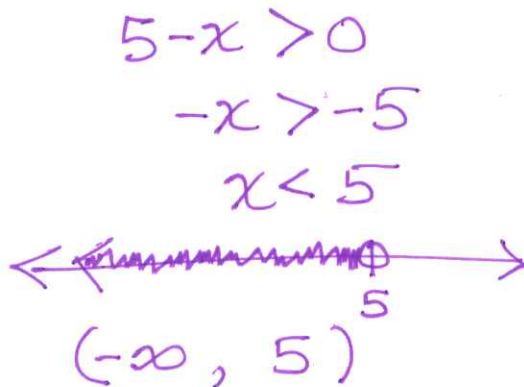
$$y = \pm \sqrt{x - 5}$$

y is NOT a function of x

10. State the domain of the following function.

$$f(x) = \frac{2x + 7}{\sqrt{5 - x}}$$

- (a) $(-\infty, 5]$
 (b) $x \neq 5$
 (c) $(-\infty, \infty)$
 (d) $(5, \infty)$
 (e) $(-\infty, 5)$



Free Response Questions: Show your work!

11. Find all real solutions of the equations below. You must use algebraic methods and SHOW YOUR WORK. ONLY exact answers will receive full credit!

4 pts. (a)

$$|x - 5| = 7$$

$$x - 5 = 7 \quad 1 \text{ pt.}$$

$$x - 5 = -7 \quad 1 \text{ pt.}$$

$$x = 12 \quad 1 \text{ pt.}$$

$$x = -2 \quad 1 \text{ pt.}$$

6 pts. (b)

$$2x^4 = 9x^2 + 5$$

$$\left. \begin{array}{l} u = x^2 \\ u^2 = x^4 \end{array} \right\} +1$$

$$2u^2 = 9u + 5$$

$$2u^2 - 9u - 5 = 0$$

$$2u^2 - 10u + 1u - 5 = 0$$

$$2u(u - 5) + 1(u - 5) = 0$$

$$(2u + 1)(u - 5) = 0$$

$$2u + 1 = 0$$

$$u - 5 = 0$$

$$u = -\frac{1}{2} + 1$$

$$u = 5 + 1$$

$$x^2 = 5$$

$$x = \pm\sqrt{5} + 2$$

eliminating
answer
+1

~~$x^2 = -\frac{1}{2}$~~
extraneous
solution

Free Response Questions: Show your work!

12. Given a circle with the endpoints of its diameter at the points $(-2, 3)$ and $(6, 3)$:

5 points → (a) Find the equation of the circle.

Center: $(\frac{-2+6}{2}, \frac{3+3}{2}) \Rightarrow (\frac{4}{2}, \frac{6}{2}) \Rightarrow (2, 3)$ (+2)

$(x-2)^2 + (y-3)^2 = r^2$ (+1) $r^2 = (-2-2)^2 + (3-3)^2$

(+3) $(x-2)^2 + (y-3)^2 = 16$ ← $r^2 = (-4)^2 + (0)^2 = 16$

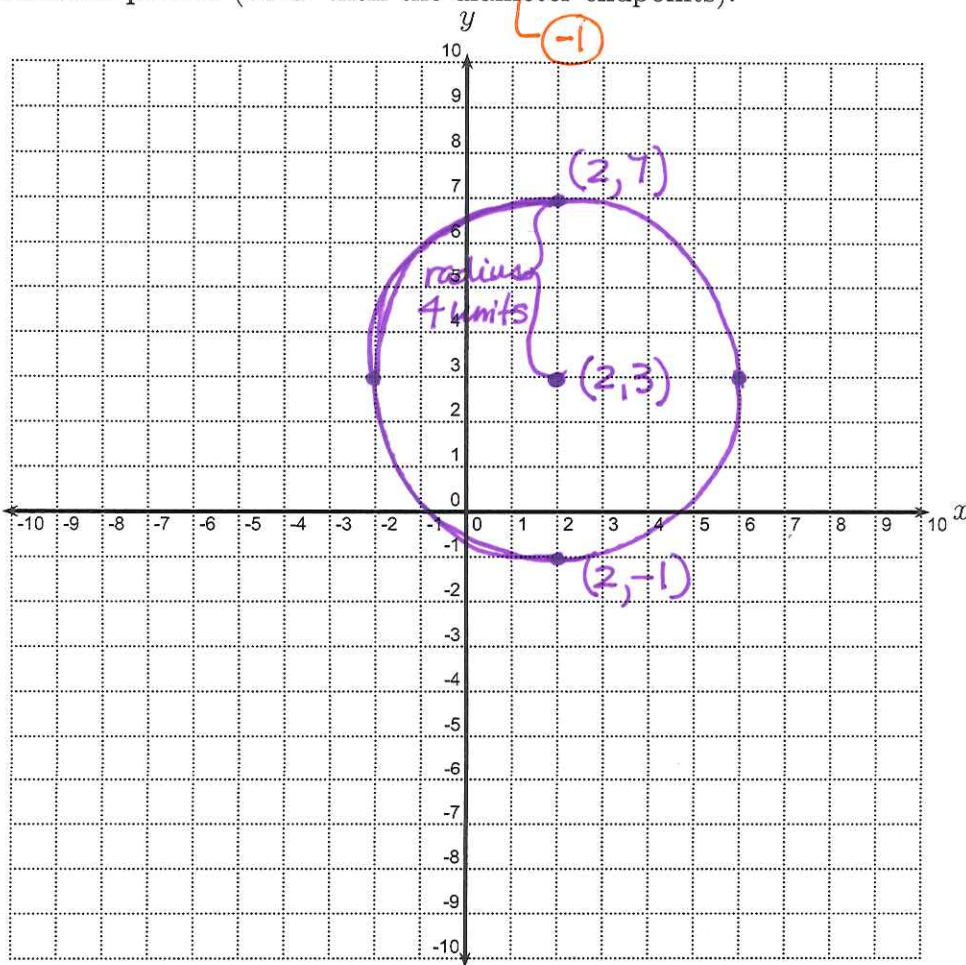
2 points → (b) Find at least two additional points on the circle (other than the diameter endpoints).

$x = 2$
 $(2-2)^2 + (y-3)^2 = 16$
 $0^2 + (y-3)^2 = 16$

$y - 3 = \pm 4$
 $y = 3 \pm 4$
 $(2, 7)$ & $(2, -1)$

(+1) (+2)

3 points → (c) Sketch a graph of the circle below. Label the center, as well as at least two additional points (other than the diameter endpoints).



Free Response Questions: Show your work!

13. According to the Center of Science in the Public Interest, the maximum healthy weight for a person who is 5 feet 5 inches tall is 150 pounds, and the maximum weight for someone 6 feet 3 inches tall is 200 pounds. The relationship between weight and height here is linear.

4 points → (a) Find a linear equation that gives the maximum healthy weight y for a person whose height is x inches over 5 feet. (Thus $x = 0$ corresponds to 5 feet, $x = 2$ corresponds to 5 feet 2 inches, etc.)

(height, weight)

(x , y)

(5, 150) (+1)

(15, 200)

$$m = \frac{200 - 150}{15 - 5} = \frac{50}{10} = 5 \quad (+1)$$

$$y - 150 = 5(x - 5) \quad (+2)$$

OR

$$y - 200 = 5(x - 15)$$

$$y = 5x + 125$$

3 pts. → (b) What is the maximum healthy weight for a person whose height is 5 feet 8 inches?

$$x = 8$$

$$y - 150 = 5(8 - 5)$$

$$y - 150 = 5(3)$$

$$y = 15 + 150$$

$$y = 165 \text{ pounds} \quad (+1)$$

3 pts. → (c) How tall is a person who is at a maximum healthy weight of 220 pounds?

$$y = 220$$

$$220 - 150 = 5(x - 5)$$

$$70 = 5(x - 5)$$

$$14 = x - 5$$

$$6 \text{ ft } 7 \text{ in.} = 19 = x \quad (+2)$$

(+1) inches over 5 feet!

Free Response Questions: Show your work!

14. Given the following parabola,

$$y^2 + 6y + 17 = 4x$$

$$* (y-k)^2 = 4p(x-h)$$

4 points → (a) Find the vertex algebraically. What are the coordinates of the vertex?

$$y^2 + 6y + \underline{9} = 4x - 17 + \underline{9}$$

$$(y+3)^2 = 4x - 8$$

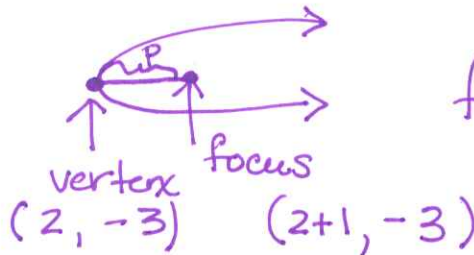
$$(y+3)^2 = 4(x-2)$$

+2 Vertex: $(\overset{+1}{2}, \overset{+1}{-3})$

2 points → (b) Find the focus. What are the coordinates of the focus?

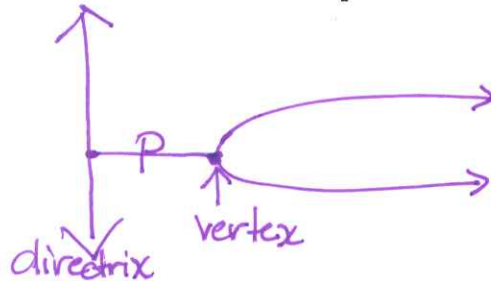
$$4p = 4$$

$$p = \underline{1}$$



focus: $(\overset{+1}{3}, \overset{+1}{-3})$

2 points → (c) Find the directrix. What is the equation of the directrix?



directrix: $x = \overset{+1}{1}$

2 points → (d) In which direction does the parabola open? Why?

$p > 0 \leftarrow \boxed{\overset{+1}{\text{right}}}$ b/c "p" is $\overset{+1}{\text{positive}}$

$y^2 \leftarrow \text{left or right}$

Free Response Questions: Show your work!

15. For the function:

$$f(x) = \begin{cases} 2x + 3 & \text{if } x < 4 \\ x^2 - 1 & \text{if } 4 \leq x \leq 10 \end{cases}$$

3 pts \rightarrow (a) Find $f(-5)$.

$$-5 < 4$$

rule (+1)

work (+1)

(+1)

$$f(-5) = 2(-5) + 3 = -10 + 3 = -7$$

3 pts \rightarrow (b) Find $f(8)$.

$$4 \leq 8 \leq 10$$

rule (+1)

work (+1)

(+1)

$$f(8) = (8)^2 - 1 = 64 - 1 = 63$$

2 pts \rightarrow (c) Find $f(15)$.

$$15 > 10$$

$$f(15) = \text{undefined}$$

2 pts \rightarrow (d) Find the domain of f .

$$\text{Domain: } (-\infty, 10]$$