

Multiple Choice Answers

Name: Solution

Section: _____

Last 4 digits of student ID #: _____

This exam has ten multiple choice questions (four points each) and five free response questions (seven points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*).
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		40
11		7
12		7
13		7
14		7
15		7
Webassign Score		75
Percentage		100

Record the correct answer to the following problem on the front page of this exam.

(1) Which one of the following is a factor of

$$2x^{60} - x^3 - 1?$$

A) $(x - 1)$

B) $(x - 2)$

C) $(x + 1)$

D) $(x + 2)$

E) None of these are factors.

$2(1)^{60} - (1)^3 - 1 = 2 - 1 - 1 = 0$ 1 is a root
 $(x-1)$ is factor

(2) Find $f(g(x))$ for

$$f(x) = \sqrt{x-2}, \quad g(x) = \frac{1}{x+1}$$

A) $\sqrt{\frac{1}{x+1}} - \sqrt{2}$

B) $\frac{1}{\sqrt{x-1}}$

C) $\sqrt{\frac{1}{x+1} - 2}$

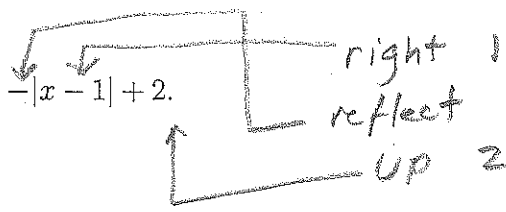
D) $\frac{1}{\sqrt{x-2} + 1}$

E) $\sqrt{\frac{x-2}{x+1}}$

$f(g(x)) = f\left(\frac{1}{x+1}\right)$
 $= \sqrt{\frac{1}{x+1} - 2}$

(3) Describe the shifting, stretching, or reflecting (flipping) across the x -axis transformations when transforming

$$f(x) = |x| \text{ to } g(x) = -|x-1| + 2.$$



A) $f(x)$ is shifted left by 2, up by 1.

B) $f(x)$ is shifted right by 2, up by 1.

C) $f(x)$ is shifted left by 2, reflected, and shifted down by 1.

D) $f(x)$ is shifted left by 1, reflected, and shifted up by 2.

E) $f(x)$ is shifted right by 1, reflected, and shifted up by 2.

Record the correct answer to the following problem on the front page of this exam.

(4) Find the value of $f(2)$ for

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < -5, \\ x & \text{for } -5 \leq x < 3, \\ 2x^2 + 1, & \text{for } 3 \leq x. \end{cases}$$

A) 3

B) $f(x)$ is undefined at $x = 2$.

C) 2

D) 9

E) 14

←
-5 ≤ 2 < 3
f(2) = 2

(5) Find the average rate of change of $f(x) = -2x^3 + 2$ as x changes from -3 to -1 .

A) 28

B) -26

C) 14

D) 52

E) 26

$$\begin{aligned} x_2 = -1 \quad x_1 = -3 \\ \frac{f(x_2) - f(x_1)}{x_2 - x_1} &= \frac{-2(-1)^3 + 2 - (-2(-3)^3 + 2)}{-1 - (-3)} \\ &= \frac{4 - 56}{2} = \frac{-52}{2} = -26 \end{aligned}$$

(6) Which one of the following functions is not one-to-one on its domain?

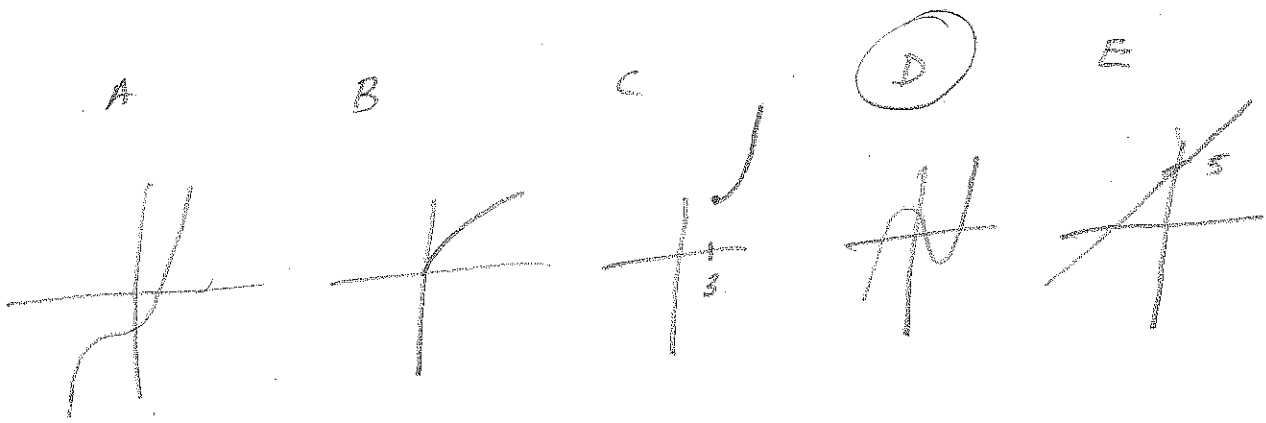
A) $f(x) = 2x^3 - 5$

B) $f(x) = \sqrt{x}$ for $x \geq 0$.

C) $f(x) = 2(x - 3)^2 + 5$ for $x > 3$.

D) $f(x) = 3x^3 - 8x + 1$

E) $f(x) = 2x + 5$



Record the correct answer to the following problem on the front page of this exam.

- (7) What is the maximum number of roots the polynomial $p(x) = -100x^4 - 200x^3 + 500x^2 - x + 200$ can have?

A) 0

B) 1

C) 2

D) 3

E) 4

deg = 4
max roots = 4

- (8) Find the domain and the range of the function shown below.

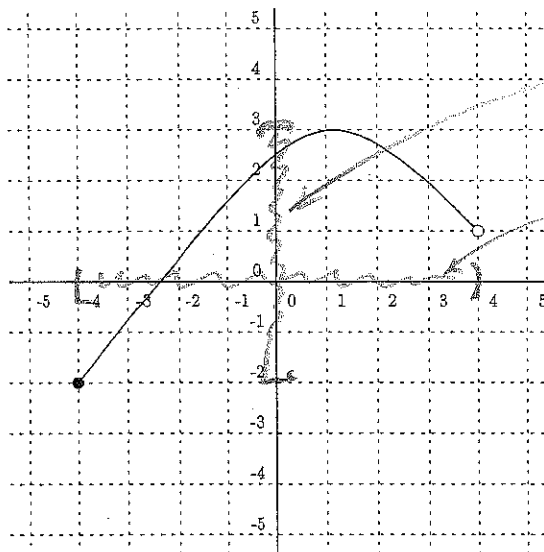
A) domain: $[-4, 4]$, range: $[-2, 3]$

B) domain: $[-4, 4)$, range: $[-2, 3]$

C) domain: $(-4, 4)$, range: $[-2, 1)$

D) domain: $[-4, 3]$, range: $[-2, 1)$

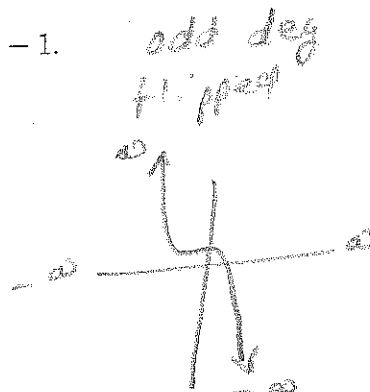
E) domain: $[-2, 3]$, range: $[-2, 4)$



Record the correct answer to the following problem on the front page of this exam.

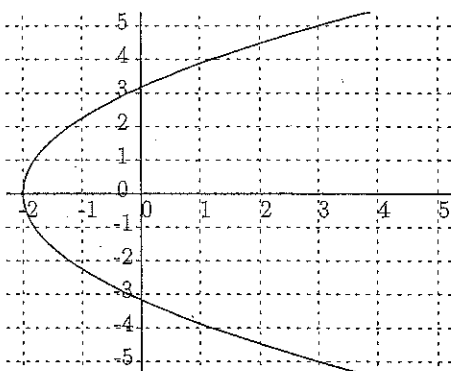
(9) Describe the end behavior of the function $f(x) = -3x^3 + 3x - 1$.

- A) $y \rightarrow \infty$ as $x \rightarrow \infty$ and $y \rightarrow -\infty$ as $x \rightarrow -\infty$
- B) $y \rightarrow \infty$ as $x \rightarrow \infty$ and $y \rightarrow \infty$ as $x \rightarrow -\infty$
- C) $y \rightarrow -\infty$ as $x \rightarrow \infty$ and $y \rightarrow \infty$ as $x \rightarrow -\infty$
- D) $y \rightarrow -\infty$ as $x \rightarrow \infty$ and $y \rightarrow -\infty$ as $x \rightarrow -\infty$
- E) The end point behavior can not be determined.



(10) For the graph below, which one of the following statements is false?

- A) The graph fails the vertical line test. *True*
- B) The graph passes the horizontal line test. *True*
- C) The graph represents x as a function of y . *True*
- D) The graph represents y as a function of x . *False*
- E) The graph is not one-to-one. *True*



Free Response Questions: Show your work!

(11) Perform the long division and write the result in the form $p(x) = D(x)Q(x) + r(x)$.

$$\frac{3x^3 + 4x^2 + x - 1}{x^2 - 1}$$

where $D(x)$ is the divisor, $Q(x)$ is the quotient, and $r(x)$ is the remainder.

$$\begin{array}{r} 3x + 4 \\ x^2 - 1 \overline{) 3x^3 + 4x^2 + x - 1} \\ \underline{-(3x^3 - 3x)} \\ 4x^2 + 4x - 1 \\ \underline{-(4x^2 - 4)} \\ 4x + 3 \end{array}$$

$$3x^3 + 4x^2 + x - 1 = \underbrace{(x^2 - 1)}_D \underbrace{(3x + 4)}_Q + \underbrace{(4x + 3)}_R$$

check: R.O.I.L.

$$3x^3 + 4x^2 - 3x - 4 + 4x + 3$$

$$= 3x^3 + 4x^2 + x - 1$$

Free Response Questions: Show your work!

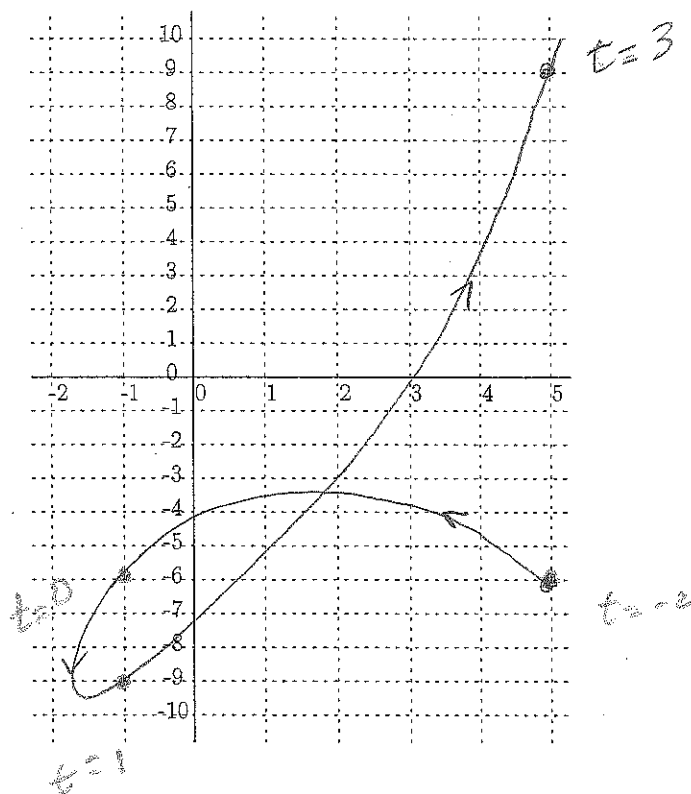
(12) A graph is given parametrically by

$$x(t) = t^2 - t - 1, y(t) = t^3 - 4t - 6, \text{ for } -2 \leq t \leq 3.$$

A) Fill in the table for the parametric equations.

t	-2	0	1	3
x	5	-1	-1	5
y	-6	-6	-9	9

B) Use a graphing calculator to sketch the graph of the parametric equations and label the points found in part A for each t value.



Free Response Questions: Show your work!

(13) Given the functions $f(x) = 2x^3 - 3$.

A) Find the inverse function $f^{-1}(x)$

$$y = 2x^3 - 3$$

$$x = 2y^3 - 3$$

$$x + 3 = 2y^3$$

$$\frac{x+3}{2} = y^3$$

$$\sqrt[3]{\frac{x+3}{2}} = y$$

$$f^{-1}(x) = \sqrt[3]{\frac{x+3}{2}}$$

B) Use the composition $f(f^{-1}(x))$ to show that the answer from part A is in fact the inverse.

$$f(f^{-1}(x)) = f\left(\sqrt[3]{\frac{x+3}{2}}\right)$$

$$= 2\left(\sqrt[3]{\frac{x+3}{2}}\right)^3 - 3$$

$$= 2\left(\frac{x+3}{2}\right) - 3$$

$$= x + 3 - 3 = x$$

$$f(f^{-1}(x)) = x \leftarrow \text{inverse}$$

Free Response Questions: Show your work!

(14) Given the functions

$$f(x) = 2x^2 + 8x + 1.$$

A) In terms of shifting and stretching, compare the graph of $f(x)$ to the graph of $f(x) = x^2$. Hint: Complete the square and write in the form $f(x) = a(x-h)^2 + k$.

$$\begin{aligned} f(x) &= 2(x^2 + 4x) + 1 \\ &= 2(x^2 + 4x + 2) + 1 - 4 \\ &= 2(x+2)^2 - 3 \end{aligned}$$

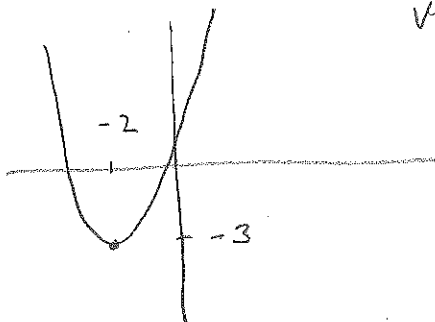
left 2
stretch 2
down 3

B) Indicate whether $f(x)$ has a minimum or a maximum and find it.

$$a = 2 > 0 \Rightarrow \text{opens up}$$

$$\text{vertex} = (-2, -3)$$

$$\text{min} = -3$$



Free Response Questions: Show your work!

(15) For the functions

$$f(x) = x^2 - 3.$$

- A) Use the difference quotient $\frac{f(x+h) - f(x)}{h}$ to get a general formula for the average rate of change for $f(x)$ in terms of x and h .

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(x+h)^2 - 3 - (x^2 - 3)}{h} = \frac{x^2 + 2xh + h^2 - 3 - x^2 + 3}{h} \\ &= \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = 2x+h \end{aligned}$$

- B) Use part A above to find the average rate of change of $f(x)$ as x changes from $x = -2$ to $x = 4$.

$$h = 4 - (-2) = 6$$

$$x = -2$$

$$2(-2) + 6 = 2$$

- C) Extra Credit (2 points)

Use part A above to estimate the instantaneous rate of change of $f(x)$ at $x = -2$.

$$h \rightarrow 0 \quad 2x+h \rightarrow 2x$$

$$2(-2) = \boxed{-4}$$