

MA 110 Algebra and Trigonometry for
 Calculus Fall 2016
 Exam 2 Tuesday, 18 October 2016

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has twelve multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

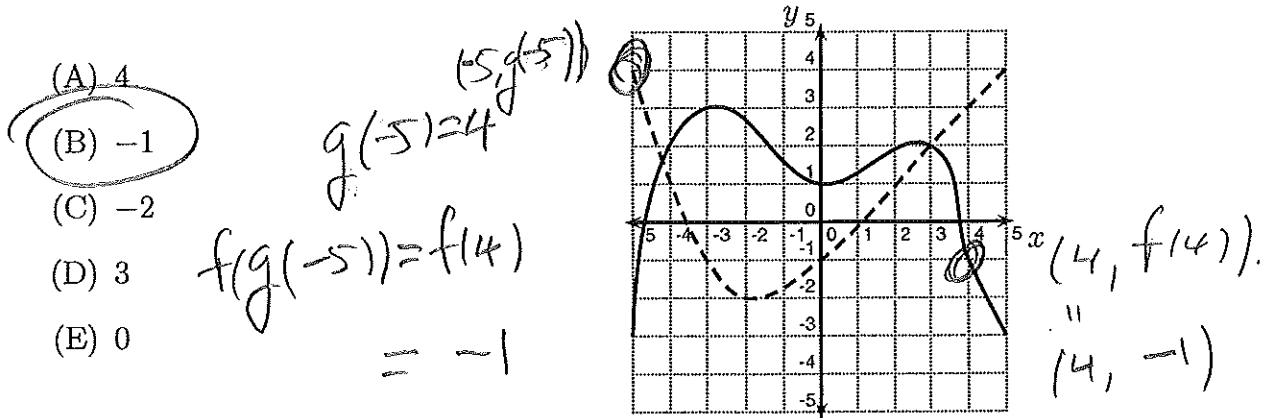
EXAMPLE	A	B	<input checked="" type="checkbox"/>	D	E
Question					
1	A	<input checked="" type="checkbox"/>	C	D	E
2	A	B	<input checked="" type="checkbox"/>	D	E
3	<input checked="" type="checkbox"/>	B	C	D	E
4	A	<input checked="" type="checkbox"/>	C	D	E
5	A	B	<input checked="" type="checkbox"/>	D	E
6	A	B	C	<input checked="" type="checkbox"/>	E
7	<input checked="" type="checkbox"/>	B	C	D	E
8	<input checked="" type="checkbox"/>	B	C	D	E
9	A	B	C	D	<input checked="" type="checkbox"/>
10	A	B	<input checked="" type="checkbox"/>	D	E
11	A	B	C	<input checked="" type="checkbox"/>	E
12	<input checked="" type="checkbox"/>	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
13		10
14		10
15		10
16		10
17		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Use the graphs to evaluate $f(g(-5))$.



2. Describe the transformations that could be applied to the graph of $f(x) = x^2$ in order to obtain the graph of $g(x) = 3x^2 - 6x + 5$.

Right 1 unit. $(x-1)^2$
 V. Stretch by factor of 3 $3(x-1)^2$
 Up 2
 5 units
 $3(x-1)^2 + 2$

$$\begin{aligned}
 g(x) &= 3x^2 - 6x + 5 = 3(x^2 - 2x + 1) + 2 \\
 &= 3(x-1)^2 - 3 + 5 \\
 &= 3(x-1)^2 + 2
 \end{aligned}$$

- (A) stretch the graph of f vertically by a factor of 3 and shift the graph up 5 units.
 (B) shrink the graph of f vertically by a factor of $1/3$ and shift the graph up 5 units.
 (C) shift the graph of f to the right 1 unit, stretch the graph vertically by a factor of 3, and shift the graph up 2 units.
 (D) shift the graph of f to the right 1 unit, stretch the graph horizontally by a factor of 3, and shift the graph up 2 units.
 (E) shift the graph of f to the left 1 unit, stretch the graph vertically by a factor of 3, and shift the graph up 8 units.

3. Given the following function. Find $f^{-1}(2)$.

$$f(x) = \frac{x+7}{3}$$

- (A) -1
 (B) $\frac{1}{2}$
 (C) 3
 (D) 1
 (E) -2

$$\begin{aligned}
 y &= \frac{x+7}{3} \\
 3y &= x+7 \\
 3y-7 &= x \\
 f^{-1}(y) &= 3y-7 \\
 f^{-1}(2) &= 3 \cdot 2 - 7 = \underline{-1} \quad 2
 \end{aligned}$$

Record the correct answer to the following problems on the front page of this exam.

4. Given a degree 3 polynomial with roots at $x = -3, 4,$ and 2 and a y -intercept of -8 , determine the end behavior of the graph.


- (A) $y \rightarrow \infty$ as $x \rightarrow \infty$ and $y \rightarrow -\infty$ as $x \rightarrow -\infty$.
 (B) $y \rightarrow -\infty$ as $x \rightarrow \infty$ and $y \rightarrow \infty$ as $x \rightarrow -\infty$.
 (C) $y \rightarrow \infty$ as $x \rightarrow \infty$ and $y \rightarrow \infty$ as $x \rightarrow -\infty$.
 (D) $y \rightarrow -\infty$ as $x \rightarrow \infty$ and $y \rightarrow -\infty$ as $x \rightarrow -\infty$.
 (E) Not enough information is given to determine the end behavior.

$$P(x) = c(x+3)(x-2)(x-4)$$

$$-8 = 24c$$

$$c = -\frac{1}{3}$$

$$P(0) = -8 = c(3)(-2)(-4) \quad P(x) = -\frac{1}{3}(x+3)(x-4)(x-2)$$

$$= -\frac{1}{3}x^3 + \dots$$


5. Find the solutions to the following inequality.

$$f(x) = (x+5)(x+2)^2(x-4) < 0$$

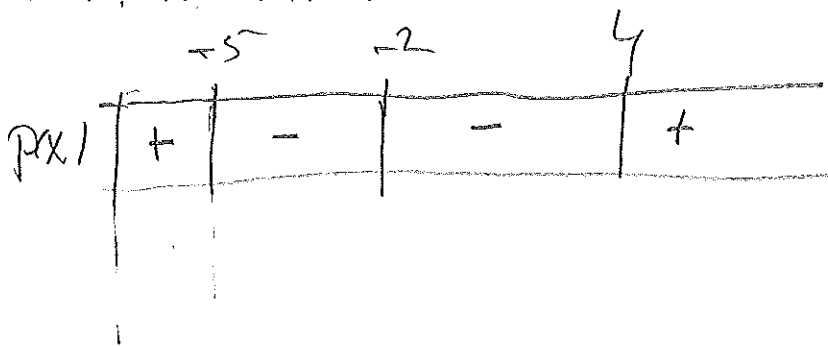
(A) $(-\infty, -5) \cup (4, \infty)$

(B) $(-5, 4)$

(C) $(-5, -2) \cup (-2, 4)$

(D) $[-5, -2) \cup (-2, 4]$

(E) $(-\infty, \infty)$



6. Suppose that the graph of f contains the point $(-5, 7)$. Find a point that must be on the graph of

$$g(x) = 2f(x+3) - 4$$

(A) $(-2, 10)$

(B) $(-5, 7)$

(C) $(-2, \frac{3}{2})$

(D) $(-8, 10)$

(E) $(8, 14)$

$$\begin{aligned} g(-8) &= 2f(-5) - 4 \\ &= 2 \cdot 7 - 4 \\ &= 10 \end{aligned}$$

$$(-8, 10)$$

Record the correct answer to the following problems on the front page of this exam.

7. Find the remainder when the following polynomial is divided by $x + 2$.

- (A) 0
 (B) -4
 (C) $x^2 - 3x + 1$
 (D) $x - 2$
 (E) -20.

$$\begin{array}{r}
 P(x) = x^3 - x^2 - 5x + 2. \\
 x^2 = 3x + 1 \\
 x+2 \overline{) x^3 - x^2 - 5x + 2} \\
 \underline{x^3 + 2x^2 - 5x} \\
 -3x^2 - 5x + 2 \\
 \underline{-3x^2 - 6x} \\
 x + 2 \\
 \underline{x + 2} \\
 0
 \end{array}$$

Remainder is 0
 Or remainder is $P(-2) = 0$

8. Describe the end behavior of the graph of the following function.

$$f(x) = \frac{7x^3 - 5x^2 + 6x - 12}{5x^3 + 3x - 8}$$

- (A) $y \rightarrow \frac{7}{5}$ as $x \rightarrow \pm\infty$
 (B) $y \rightarrow 7$ as $x \rightarrow \pm\infty$
 (C) $y \rightarrow 0$ as $x \rightarrow \pm\infty$
 (D) $y \rightarrow \infty$ as $x \rightarrow \pm\infty$
 (E) $y \rightarrow \frac{5}{7}$ as $x \rightarrow \pm\infty$

Horiz. asymptote at $y = \frac{7}{5}$.

9. Solve the following equation and express the answer in standard complex form.

$$2x^2 - 8x = -14$$

- (A) $4 \pm \sqrt{12}i$
 (B) $-2 \pm \sqrt{3}i$
 (C) $4 \pm \sqrt{44}i$
 (D) $2 \pm \sqrt{11}i$
 (E) $2 \pm \sqrt{3}i$

$$\begin{aligned}
 x^2 - 4x &= -7 \\
 x^2 - 4x + 4 &= -7 + 4 \\
 (x-2)^2 &= -3 \\
 x - 2 &= \pm \sqrt{-3} = \pm \sqrt{3}i \\
 x &= 2 \pm \sqrt{3}i
 \end{aligned}$$

Record the correct answer to the following problems on the front page of this exam.

10. Solve for x exactly.

- (A) $x = 11$
- (B) $x = 25$
- (C) $x = -11$
- (D) $x = -14$
- (E) No solution

$$\begin{aligned} \log_5(3 - 2x) &= 2 \\ 5^{\log_5(3 - 2x)} &= 5^2 \\ 3 - 2x &= 25 \\ 3 - 2x &= 25 \\ -3 & \quad -3 \\ -2x &= 22 \\ x &= -11 \end{aligned}$$

11. Simplify the following expression.

- (A) $x^{\frac{4}{3}}y^{-\frac{4}{3}}$
- (B) x^4y^{-8}
- (C) $x^{\frac{5}{2}}y^{\frac{3}{4}}$
- (D) $x^{\frac{3}{2}}y^{-\frac{7}{3}}$
- (E) $x^4y^{\frac{2}{3}}$

$$\begin{aligned} \frac{\sqrt[3]{x^5}\sqrt{y^{-3}}}{\sqrt[6]{xy^5}} &= \frac{x^{5/3}y^{-3/2}}{x^{1/6}y^{5/6}} \\ &= x^{5/3 - 1/6} y^{-3/2 - 5/6} \\ &= x^{9/6} y^{-14/6} = x^{3/2} y^{-7/3} \end{aligned}$$

12. If \$2,500 is deposited into a bank account that compounds continuously with an annual interest rate of $r = 3.2\%$, how much will the account be worth in 6 years?

- (A) \$3,029.18
- (B) \$20,656.92
- (C) \$15,487.76
- (D) \$17,052.40
- (E) \$2,423.34

Amount at time t is $2500 \cdot e^{0.032t}$
 After 6 years, this is
 $2500 \cdot e^{6 \cdot 0.032}$
 $= \underline{\underline{\$3029.18}}$

Free Response Questions: Show your work!

13. The profit made from the deluxe pinwheel package at Miss Lola's Pinwheel Manufacturing Company is given by the following function, where x represents the number of deluxe packages sold.

$$p(x) = 1750x - 5x^2 - 48000$$

- (a) How much profit is made if 50 packages are sold?

$$\begin{aligned} P(50) &= 1750(50) - 5(50)^2 - 48000 \\ &= 87500 - 12500 - 48000 \\ &= \$27,000 \end{aligned}$$

- (b) What will the maximum profit be?

$$\begin{aligned} & -5x^2 + 1750x - 48000 \\ x &= \frac{-b}{2a} = \frac{-1750}{2(-5)} = 175 \end{aligned}$$

$$P(175) = 1750(175) - 5(175)^2 - 48000 = 105,125$$

Max profit is \$105,125

- (c) How many packages can Miss Lola sell before the company starts to lose money?

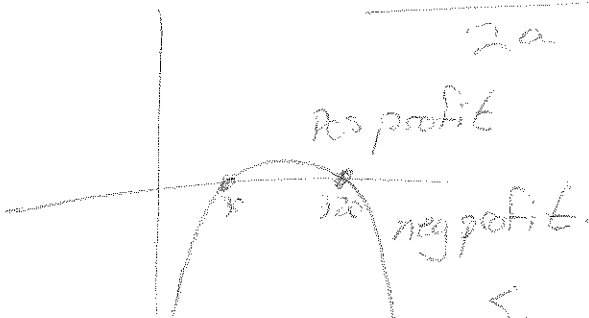
Find largest x -int

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1750 \pm \sqrt{1750^2 - 4(-5)(-48000)}}{2(-5)}$$

$$= 175 \pm 145$$

$$\Rightarrow x = 30, x = 320$$

So, Miss Lola can sell 320 packages before losing money.



Free Response Questions: Show your work!

14. Given the following two functions, find each of the following.
Make certain to **simplify** your answers.

$$f(x) = 7 - x^2$$

$$g(x) = 2x - 5$$

(a) $(f \circ g)(x)$ $f(2x-5) = 7 - (2x-5)^2 = 7 - (4x^2 - 20x + 25)$

$$\boxed{-4x^2 + 20x - 18}$$

(b) $g(g(x))$ $g(2x-5) = 2(2x-5) - 5 = 4x - 10 - 5 =$

$$\boxed{4x - 15}$$

(c) $f(5 - 3g(-2))$ $g(-2) = 2(-2) - 5 = -4 - 5 = -9$

$$f(5 - 3(-9)) = f(5 + 27) = f(32) =$$

$$7 - 32^2 =$$
$$7 - 1024 = \boxed{-1017}$$

Free Response Questions: Show your work!

15. Given the following polynomial, determine whether each of the statements are either **TRUE** or **FALSE**. Write your answer on the line preceding each statement. Justify your reasoning below each statement.

$$p(x) = (x - 3)^3(x + 2)(x - 4)^2(x + 7)$$

- (a) False The graph will cross the x -axis at $x = 2$.

$x+2$ is a factor, so there is a root at $x = -2$, not $x = 2$

- (b) True $y \rightarrow \infty$ as $x \rightarrow \infty$.

polynomial is of degree 7 and has leading coefficient ≥ 1

- (c) True The graph will touch the x -axis at $x = 4$.

Factor of $(x - 4)$ to an even power, so graph will touch and not cross

- (d) False $y \rightarrow \infty$ as $x \rightarrow -\infty$.

polynomial is of degree 7 and has leading coefficient $= 1$

- (e) False The graph will have 3 local extrema.

Has 4 local extrema:

- between -7 and -2
- between -2 and 3
- between 3 and 4
- at 4

Free Response Questions: Show your work!

16. Considering the graph of the following function, determine the intercepts, asymptotes, and holes, if any. You MUST show your work indicating how you arrived at your answers. It is NOT sufficient to use a graph from your calculator!

$$f(x) = \frac{2x^2 + 4x - 6}{x^2 - 1}$$

- (a) The x -intercept(s) are

x -int. happen when $f(x) = 0$

$$0 = \frac{2(x^2 + 2x - 3)}{(x+1)(x-1)} = \frac{2(\cancel{x-1})(x+3)}{(\cancel{x-1})(x+1)} \Rightarrow \begin{aligned} 0 &= x + 3 \\ -3 &= x \end{aligned}$$

$$\boxed{(-3, 0)}$$

- (b) The y -intercept is

y -int. happen when $x = 0$

$$f(0) = \frac{2(0)^2 + 4(0) - 6}{0^2 - 1} = \frac{-6}{-1} = 6$$

$$\boxed{(0, 6)}$$

- (c) The vertical asymptote(s) are

V.A. \Rightarrow zeros of denominator; NOT zeros of numerator

$$x + 1 = 0$$

$$\boxed{x = -1}$$

- (d) The horizontal asymptote(s) are

H.A. \Rightarrow ratio of leading terms

$$\frac{2x^2}{x^2} = 2 \Rightarrow y \rightarrow 2 \text{ as } x \rightarrow \pm\infty$$

$$\boxed{y = 2}$$

- (e) The hole(s) are

holes \Rightarrow zeros of BOTH numerator & denominator

$$x - 1 = 0$$

$$x = 1$$

$$\boxed{(1, 4)}$$

$$\frac{2(\cancel{x}+3)}{(1+1)} = 4$$

Free Response Questions: Show your work!

17. Consider the function

$$f(x) = \ln(3x + 7)$$

(a) Find the domain of f . Give your answer in interval notation.

$$\begin{aligned} 3x + 7 &> 0 \\ 3x &> -7 \\ x &> -7/3 \end{aligned} \quad \boxed{(-7/3, \infty)}$$

(b) Find the range of f . Give your answer in interval notation.

$$\boxed{(-\infty, \infty)}$$

(c) Find $f^{-1}(x)$

$$\begin{aligned} y &= \ln(3x + 7) \\ x &= \ln(3y + 7) \\ e^x &= 3y + 7 \end{aligned} \quad \begin{aligned} e^x - 7 &= 3y \\ \frac{e^x - 7}{3} &= y \\ \frac{e^x - 7}{3} &= f^{-1}(x) \end{aligned}$$

(d) Find the domain of f^{-1} . Give your answer in interval notation.

$$\text{Range of } f \Rightarrow \boxed{(-\infty, \infty)}$$

(e) Find the range of f^{-1} . Give your answer in interval notation.

$$\text{Domain of } f \Rightarrow \boxed{(-7/3, \infty)}$$