Math 110: Algebra for Trig and Calculus
Tuesday, October 17, 2017
Exam 2 Fall 2017

Name: KEY

Section: _____

Last 4 Digits of Student ID #: _____

This exam has twelve multiple choice questions (5 points each), five true/false questions (2 points each), and three free response questions (10 points each). Additional blank sheets are available for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has scientific or graphing capabilities.

On the multiple choice and true/false choice problems:

1. You must give your final answer in the multiple choice and true/false answer boxes on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated in the answer boxes.

On the free response problems:

1. Write your solutions neatly in the space below the question (unsupported answers may not receive credit). You are not expected to write your solution next to the statement of the question.
2. Give exact answers, rather than decimal approximations (unless otherwise stated).

Multiple Choice Answers

EXAMPLE	A	B	C	D	E
Question					
1	A	B	C	**D**	E
2	A	**B**	C	D	E
3	A	**B**	C	D	E
4	A	B	**C**	D	E
5	**A**	B	C	D	E
6	**A**	B	C	D	E
7	A	**B**	C	D	E
8	A	B	**C**	D	E
9	A	B	C	D	**E**
10	A	B	**C**	D	E
11	**A**	B	C	D	E
12	**A**	B	C	D	E

True/False Choice Answers

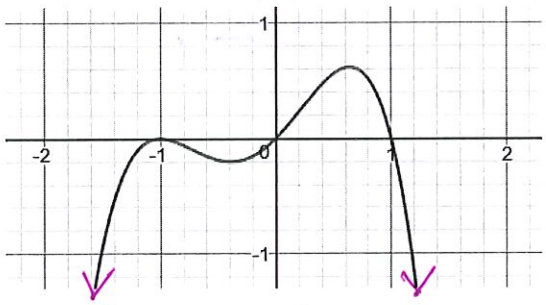
Question		
13	**T**	F
14	T	**F**
15	T	**F**
16	T	**F**
17	T	**F**

Exam Scores

Question	Score	Total
MC		60
TF		10
18		10
19		10
20		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. The graph of a polynomial function is shown below. Determine whether its degree is even or odd and whether its leading coefficient is positive or negative.
   (a) Odd degree, positive leading coefficient
   (b) Odd degree, negative leading coefficient
   (c) Even degree, positive leading coefficient
   (d) Even degree, negative leading coefficient
   (e) None of the other choices.



⇒ end behavior in the same direction indicates EVEN degree
⇒ end behavior down indicates NEGATIVE coefficient.

2. Determine the vertex of the parabola $f(x) = -(x-\pi)^2 - \sqrt{3}$.
   (a) $(-\pi, -\sqrt{3})$
   (b) $(\pi, -\sqrt{3})$
   (c) $(\sqrt{3}, \pi)$
   (d) $(-\sqrt{3}, \pi)$
   (e) None of the other choices.

Any quadratic written in $f(x)=a(x-h)^2+k$ "standard (vertex) form" has its vertex at (h,k)

$f(x) = -(x-\boxed{\pi})^2 \boxed{-\sqrt{3}}$
$f(x) = a(x-\boxed{h})^2 + \boxed{k}$

$h=\pi$ and $k=-\sqrt{3}$

3. Compute the remainder when $x^7 - 15x^3 + 11$ is divided by $x+2$.
   (a) -15
   (b) 3
   (c) 19
   (d) 45
   (e) None of the other choices.

Thm: If $\frac{P(x)}{x-c}$, then $P(c)$ = remainder

So, $\frac{x^7-15x^3+11}{x-(-2)} \Rightarrow P(-2)$ = remainder

$(-2)^7 - 15(-2)^3 + 11$
$-128 - 15(-8) + 11$
$-128 + 120 + 11$
$-8 + 11$
$\boxed{3}$

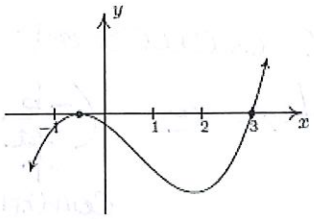
Record the correct answer to the following problems on the front page of this exam.

4. Which of the following graphs match the function $p(x) = x^3(x+2)^2$?

x^3 → $x=0$ root w/ odd multiplicity, so graph should cross at $x=0$

$(x+2)^2$ → $x+2=0$, $x=-2$ root w/ even multiplicity, so graph should ONLY touch

A)



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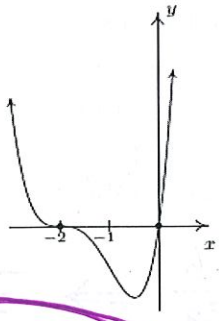
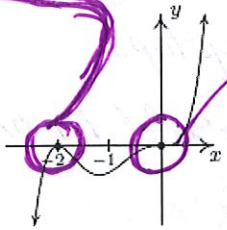
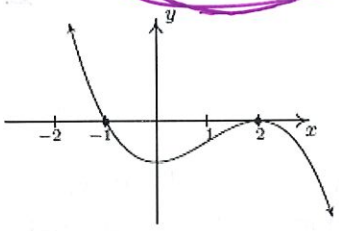


C)



D)



5. Which of the following graphs match the function $f(x) = \frac{x}{x-1}$?

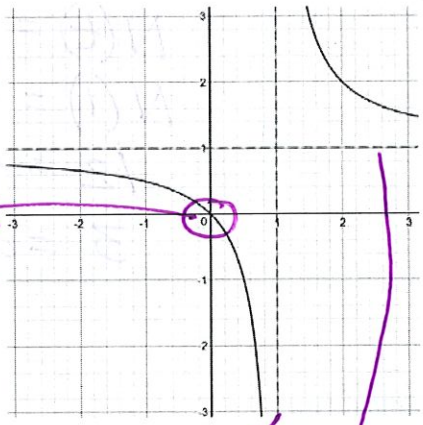
y-int when $x=0$, so $y = \frac{0}{0-1} = 0$

$(0,0)$ is y-int.

Vertical asymptote @ $x-1=0$, $x=1$

Horizontal asymptote @ $y = \frac{1x}{1x} = 1$, $y=1$

A)



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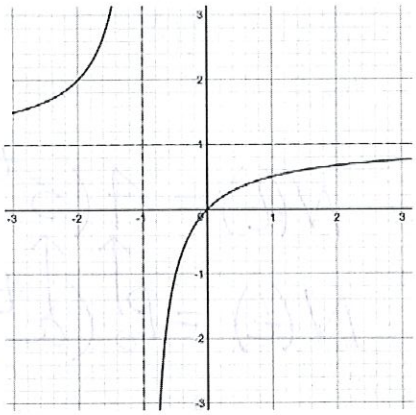

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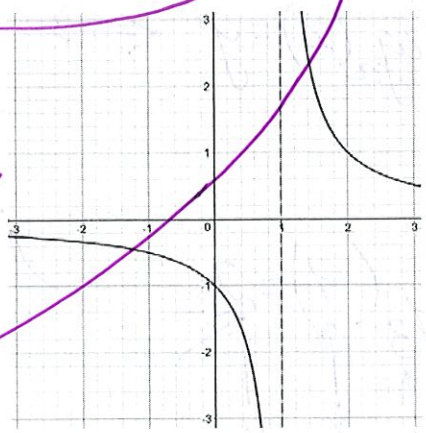
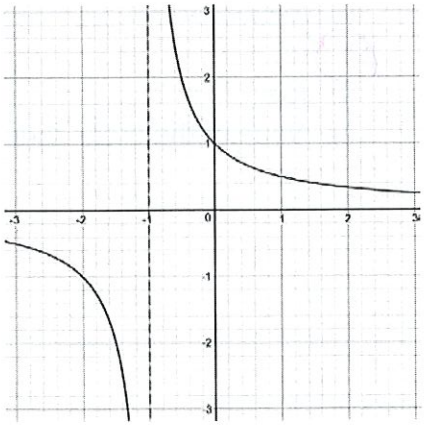


D)



Record the correct answer to the following problems on the front page of this exam.

6. A company determines that the profit from selling x laptops is modeled by the function $P(x) = -x^2 + 120x - 2600$. How many laptops should be sold to maximum profit?
   (a) 60 laptops
   (b) 120 laptops
   (c) 1000 laptops
   (d) 2600 laptops
   (e) None of the other choices.

⇒ max occurs at vertex
⇒ vertex @ $\left(\frac{-b}{2a}, P\left(\frac{-b}{2a}\right)\right)$
↑ conditions for max; ↑ actual max

"h" $= \frac{-b}{2a} \Rightarrow \frac{-120}{2(-1)} = \frac{-120}{-2} = 60$ laptops

↳ sold in order to get max profit!

7. A bacterial culture begins with 4 cells and then triples each day later. Determine a function $N(t)$ for the number of cells in the culture t days later.
   (a) $N(t) = 3(4^t)$
   (b) $N(t) = 4(3^t)$
   (c) $N(t) = 3\left(\frac{1}{4^t}\right)$
   (d) $N(t) = 4\left(\frac{1}{3^t}\right)$
   (e) None of the other choices.

$P_0 = 4$ ↓ $N(1) = 4 \cdot 3 = 12$

$N(t) = P_0 a^t$

$N(t) = 4a^t$

$N(1) = 4a^1$

$12 = 4a$

$3 = a$

$N(t) = 4(3^t)$

$N(t) = P_0(a^t)$

8. Convert the equation $3^2 = 9$ into logarithmic form.
   a) $\log_2(3) = 9$
   b) $\log_9(2) = 3$
   c) $\log_3(9) = 2$
   d) $\log_9(3) = 2$
   e) None of the other choices.

$\log_a(x) = y \iff a^y = x$

$3^2 = 9$

$\log_3(9) = 2$

$a \quad x \quad y$

Record the correct answer to the following problems on the front page of this exam.

9. Evaluate the expression $\log_{64}(8)$.
   a) 0
   b) 1
   c) 2
   d) 3
   e) None of the other choices.

$\log_{64}(8) = x$ ← make log equation

$64^x = 8$ ← rewrite equation in exponential form

$64^{(1/2)} = 8$

$\boxed{x = 1/2}$

10. Which of the following expands the expression $\ln\left(\frac{\sqrt{x}}{y^3 z}\right)$ as a sum/difference of multiples of logarithms?
   a) $2\ln(x) - 3\ln(y) + \ln(z)$
   b) $\frac{1}{2}\ln(x) - 3\ln(y) + \ln(z)$
   c) $\frac{1}{2}\ln(x) - 3\ln(y) - \ln(z)$
   d) $2\ln(x) - 3\ln(y) - \ln(z)$
   e) None of the other choices.

$\ln\left(\frac{\sqrt{x}}{y^3 z}\right) = \ln(\sqrt{x}) - [\ln(y^3 z)]$

$= \ln(x^{1/2}) - [\ln(y^3) + \ln(z)]$

$= \ln(x^{1/2}) - \ln(y^3) - \ln(z)$

$= \frac{1}{2}\ln(x) - 3\ln(y) - \ln(z)$

11. Which of the following combines the expression $-3\log(x) - 2\log(y)$ as a single logarithm?
   a) $\log\left(\frac{1}{x^3 y^2}\right)$
   b) $\log(x^3 y^2)$
   c) $\log\left(\frac{x^3}{y^2}\right)$
   d) $\log\left(\frac{y^2}{x^3}\right)$
   e) None of the other choices.

$-3\log(x) - 2\log(y)$

$\log(x^{-3}) + \log(y^{-2})$

$\log(x^{-3} y^{-2})$

$\log\left(\frac{1}{x^3 y^2}\right)$

Record the correct answer to the following problems on the front page of this exam.

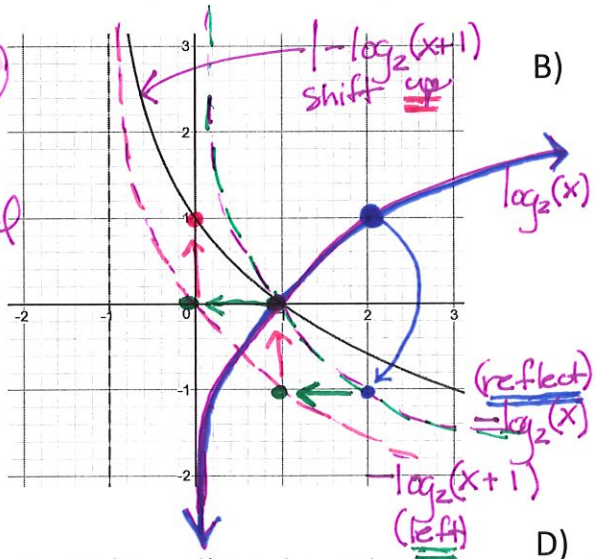
12. Which of the following graphs match the function $f(x) = 1 - \log_2(x+1)$?

$1-\log_2(x+1)$ A)

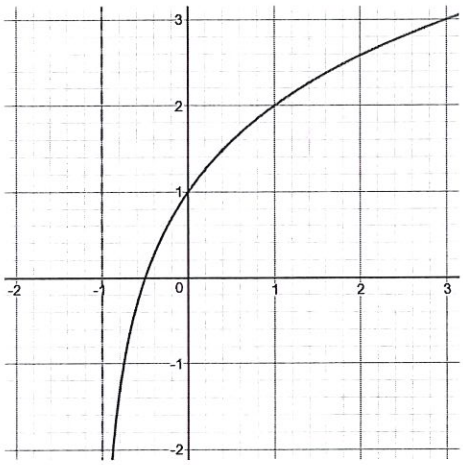
vertical shift up 1 unit

horizontal shift left 1 unit

reflect across x-axis



B)



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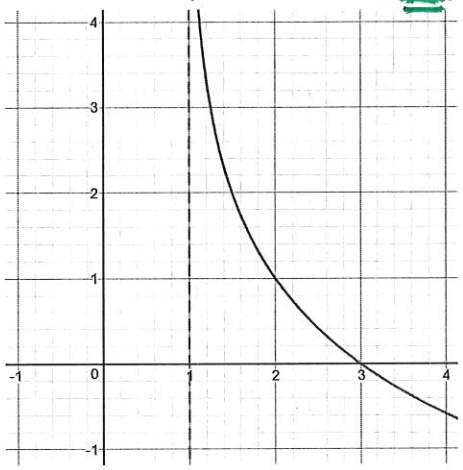
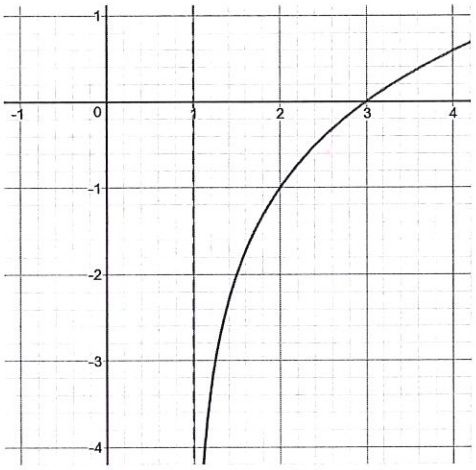


D)



For questions 13-17, determine whether each statement is true or false.

Thm: If P(x), D(x), then D(x) is a factor if and only if remainder is 0.

13. The remainder in polynomial division is 0 when the divisor is a factor of the dividend. TRUE

14. A polynomial of degree 3 always has ~~exactly~~ at most 3 real and distinct roots. FALSE

15. The domain of $f(x) = \log(x)$ is ~~all real numbers~~ $x > 0$. FALSE

16. For all real numbers A and B: $\log(A+B) = \log(A) + \log(B)$ FALSE

$\log(AB)$ =

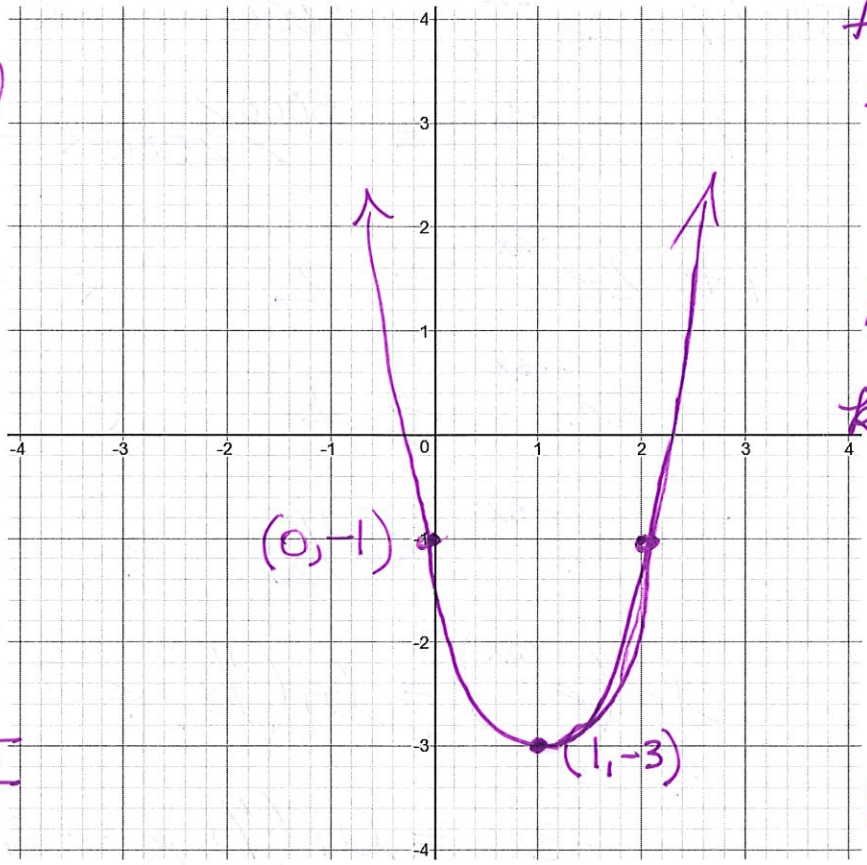
17. ~~$\ln(0) = 1$~~ FALSE

$\ln(1) = 0 \iff e^0 = 1$

Free Response Questions: Show your work!!

18. Graph the following quadratic function. Compute the **coordinates** of the vertex and the *x*- and *y*-intercepts, if any exist. Also, determine the domain and range and write the answer in **interval notation**. Record your answers in the spaces provided below. (If no *x*- or *y*-intercepts exist, write NONE in the space provided.)

$$f(x) = 2x^2 - 4x - 1$$



* x-int. when $y = 0$

$0 = 2x^2 - 4x - 1$

$+2 = 2(x^2 - 2x + 1)$

$3 = 2(x-1)^2$

$3/2 = (x-1)^2$

$\pm\sqrt{3/2} = x - 1$

$1 \pm \sqrt{3/2} = x$

OR

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-1)(2)}}{2(2)}$

$= \frac{4 \pm \sqrt{16+8}}{4}$

$= \frac{4 \pm \sqrt{24}}{4} = \frac{4 \pm 2\sqrt{6}}{4}$

$= 1 \pm \frac{\sqrt{6}}{2}$

* vertex

$f(x) = 2(x^2 - 2x + 1) - 1 - 2$

$= 2(x-1)^2 - 3$

(h, k)

OR

$h = -\frac{b}{2a} = \frac{-(-4)}{2(2)} = 1$

$k = f(-\frac{b}{2a})$

$= f(1)$

$= 2(1)^2 - 4(1) - 1$

$= 2 - 5$

$= -3$

$(h, k) = (1, -3)$

Vertex Coordinates: $(1, -3)$ (+1) Domain: $(-\infty, \infty)$ (+1)

x-intercept(s) Coordinates: $(1+\sqrt{\frac{3}{2}}, 0)$ $(1-\sqrt{\frac{3}{2}}, 0)$ (+2) Range: $[-3, \infty)$ (+1)

y-intercept Coordinates: $(0, -1)$ (+1)

* y-int. when $x = 0$

$f(x) = y = 2(0)^2 - 4(0) - 1$

$y = 0 - 0 - 1$

$y = -1$

graph:

(+1) → parabolic shape

(+1) → open up

(+1) → vertex

(+1) → y-intercept

Free Response Questions: Show your work!!

19. Solve the following equation.

$$2500 = \frac{5000}{1 + 2e^{-3x}}$$

$2500(1 + 2e^{-3x}) = 5000$ ← multiply (+1)

$1 + 2e^{-3x} = 5000/2500$ ← divide (+1)

$1 + 2e^{-3x} = 2$ ← simplify (+1)

$2e^{-3x} = 1$ ← subtract (+1)

$e^{-3x} = 1/2$ ← divide (+1)

$\ln(e^{-3x}) = \ln(1/2)$ ← take ln (+1)

$-3x = \ln(1/2)$ ← simplify (+1)

$x = \frac{-\ln(1/2)}{3}$ ← final answer (+1)

All notation (+1)

Free Response Questions: Show your work!!

20. Solve the following equation.

$$\log_3(x-4) + \log_3(x+4) = 2$$

$\log_3(x-4) + \log_3(x+4) = 2$ ← log property (+2)

$\log_3[(x-4)(x+4)] = 2$ ← simplify (+1)

$\log_3(x^2-16) = 2$ ← exponential form (+2)

$3^2 = x^2 - 16$ ← simplify (+1)

$9+16 = x^2$

$25 = x^2$

$\pm 5 = x$ ← both answers (+2)

$X=5$ ↑ correct answer choice (+1)

~~$X=-5$~~ not a viable answer!

All notation (+1)