MA 110 Algebra and Trigonometry for Calculus — Spring 2017

Exam 2 — Tuesday, 7 March 2017

Name: KEY

Section: ______________________

Last 4 digits of student ID #: ________

This exam has twelve multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

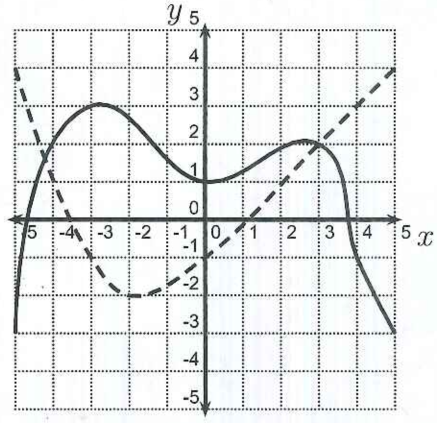
EXAMPLE	A	B	C	D	E
Question					
1	A	B	~~C~~	D	E
2	A	B	C	~~D~~	E
3	A	B	C	D	~~E~~
4	~~A~~	B	C	D	E
5	A	B	C	D	~~E~~
6	~~A~~	B	C	D	E
7	A	B	C	~~D~~	E
8	A	B	C	~~D~~	E
9	A	~~B~~	C	D	E
10	A	~~B~~	C	D	E
11	A	B	C	D	~~E~~
12	A	B	~~C~~	D	E

Exam Scores

Question	Score	Total
MC		50
13		10
14		10
15		10
16		10
17		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Use the graphs to evaluate $g(f(0))$.



(A) 4

(B) -2

(C) 0

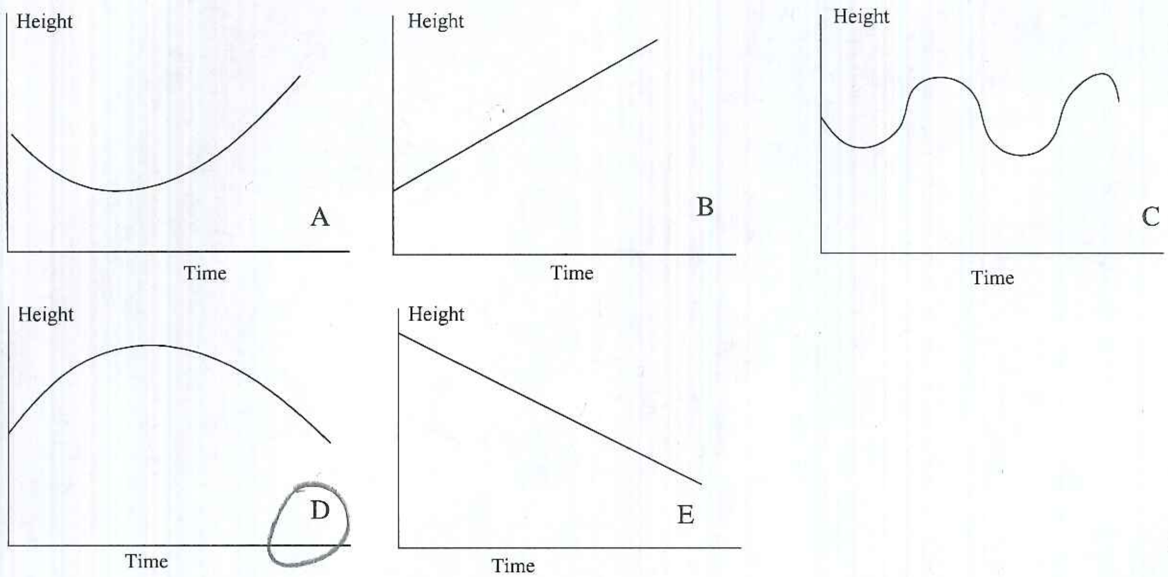
(D) 3

(E) -1

$f(0) = 1$

$g(1) = 0$

2. Consider the five graphs below. Which of these best describes the height of a ball thrown up from the top of a building as a function of time?



3. Given $f(x) = |x|$, find a rule for $g(x)$ whose graph is obtained by transforming the graph of f in the following way:

shift the graph to the left by 3 units, stretch the graph vertically by a factor of 2, and shift the graph down 4 units

(a) $g(x) = 2|x - 3| + 4$

(b) $g(x) = \frac{1}{2}|x + 3| - 4$

(c) $g(x) = \frac{1}{2}|x - 3| + 4$

(d) $g(x) = 2|x - 3| - 4$

(e) $g(x) = 2|x + 3| - 4$

vertical stretch

left 3 units

$2 f(x + 3) - 4$

down 4 units

$2|x+3| - 4$

Record the correct answer to the following problems on the front page of this exam.

4. Given the following functions, find the value of $g(f(2)+3)$.

$$g(t) = t^2 - t \qquad \text{and} \qquad f(x) = 1 + x$$

(a) 30
(b) 2
(c) 40
(d) 18
(e) 8

$f(2) = 1 + 2 = 3$

$g(f(2)+3) = g(3+3) = g(6)$

$g(6) = 6^2 - 6 = 36 - 6 = 30$

5. Given the following function, find $f^{-1}(x)$.

$$f(x) = \frac{x-5}{x+8}$$

(a) $f^{-1}(x) = \dfrac{x-5}{x+8}$
(b) $f(x)$ is not a one-to-one function.
(c) $f^{-1}(x) = \dfrac{8x-5}{x-1}$
(d) $f^{-1}(x) = \dfrac{-8x-5}{x}$
(e) $f^{-1}(x) = \dfrac{8x+5}{1-x}$

$x = \dfrac{y-5}{y+8}$

$x(y+8) = y-5$

$xy + 8x = y - 5$

$xy - y = -8x - 5$

$y(x-1) = -1(8x+5)$

$y = \dfrac{-(8x+5)}{x-1} = \dfrac{8x+5}{1-x}$

6. Given the following function, find $g^{-1}(2)$

$$g(x) = \frac{1}{3}x^3 + 11.$$

(a) -3
(b) 2
(c) $-\frac{1}{3}$
(d) $\frac{3}{2}$
(e) 3

$x = \frac{1}{3}y^3 + 11$

$x - 11 = \frac{1}{3}y^3$

$3(x-11) = y^3$

$3x - 33 = y^3$

$\sqrt[3]{3x-33} = y = g^{-1}$

$g^{-1}(2) = \sqrt[3]{3(2)-33}$
$= \sqrt[3]{-27}$
$= -3$

Record the correct answer to the following problems on the front page of this exam.

7. Find the equation of the unique quadratic function with a vertex at the point $(2, 4)$ and which passes through the point $(0, -2)$

   (a) $f(x) = -\frac{2}{3}(x+2)^2 + 4$
   (b) $f(x) = -2(x-2)^2 + 4$
   (c) $f(x) = \frac{3}{2}(x-4)^2 - 2$
   (d) $f(x) = -\frac{3}{2}(x-2)^2 + 4$
   (e) There is not enough information given to answer the question.

$f(x) = a(x-2)^2 + 4$

$-2 = a(0-2)^2 + 4$

$-2 = 4a + 4$

$-6 = 4a \quad a = \frac{-6}{4} = -\frac{3}{2}$

$f(x) = -\frac{3}{2}(x-2)^2 + 4$

8. Describe the end behavior of the graph of the following polynomial function.

$$Q(x) = -55x^{100} + 15x^{75} - 3$$

negative
even

   (a) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
   (b) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
   (c) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
   (d) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
   (e) $y \to -55$ as $x \to \infty$ and $y \to -55$ as $x \to -\infty$

$\swarrow \quad \searrow$

9. Given the following functions, find the remainder when $f(x)$ is divided by $g(x)$.

$$f(x) = 3x^{90} - 3x^{70} + 3x^{50} - 5x^{35} - 2x^{16} + 3 \qquad \text{and} \qquad g(x) = x + 1$$

   (a) -1 $f(-1) = 3 - 3 + 3 + 5 - 2 + 3$
   (b) 9 $= 9$
   (c) 4
   (d) -2
   (e) 2

Record the correct answer to the following problems on the front page of this exam.

10. Give the domain of the following function.

$$h(x) = \frac{x^7 + 2x^3 - 5x - \pi}{2x^4 + 16x}$$

(a) $(-\infty, \infty)$
(b) $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$
(c) $(-\infty, -2) \cup (0, \infty)$
(d) $(-\infty, -2) \cup (-2, \infty)$
(e) $(-\infty, 0) \cup (0, \infty)$

$2x^4 + 16x \neq 0$

$2x(x^3 + 8) \neq 0$

$2x \neq 0$ $\quad$ $x^3 + 8 \neq 0$

$\boxed{x \neq 0}$ $\quad$ $x^3 \neq -8$

$x \neq \sqrt[3]{-8}$

$\boxed{x \neq -2}$

$(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$

11. Solve the inequality

$$x^2 - 2x \geq 8$$

(a) $(-2, 4)$
(b) $[-2, 4]$
(c) $(-\infty, -4] \cup [2, \infty)$
(d) $[-4, 2]$
(e) $(-\infty, -2] \cup [4, \infty)$

$x^2 - 2x - 8 \geq 0$

$(x-4)(x+2) \geq 0$

$x - 4 = 0$ $\quad$ $x + 2 = 0$

$x = 4$ $\quad$ $x = -2$

>0 $\quad$ <0 $\quad$ >0

-5 $\quad$ -2 $\quad$ 0 $\quad$ 4 $\quad$ 5

$(-\infty, -2] \cup [4, \infty)$

12. Multiply and simplify.

$$\left(x^{\frac{3}{2}} - y^{\frac{3}{2}}\right)\left(x^{\frac{3}{2}} + y^{\frac{3}{2}}\right)$$

(a) $x - y$
(b) $x^{\frac{3}{2}} - y^{\frac{3}{2}}$
(c) $x^3 - y^3$
(d) $x^{\frac{9}{4}} - y^{\frac{9}{4}}$
(e) $x + y$

$x^{3/2} x^{3/2} + x^{3/2} y^{3/2} - x^{3/2} y^{3/2} - y^{3/2} y^{3/2}$

$x^{3/2 + 3/2} - y^{3/2 + 3/2}$

$x^{6/2} - y^{6/2}$

$x^3 - y^3$

Free Response Questions: Show your work!

13. Given the function g below, answer each of following questions.

$$g(x) = -3x^2 - 18x - 32$$

(a) Use "completing the square" to write g in standard (*vertex*) form. What are the coordinates of the vertex?

$-3x^2 - 18x - 32$

$-3(x^2 + 6x) - 32$

$-3(x^2 + 6x + \underline{9}) - 32 + \underline{27}$

$-3(x+3)^2 - 5$

vertex: $(-3, -5)$

(b) What is the **absolute maximum** of g?

g(x) is a parabola opening down, so the max is the vertex.

absolute maximum: -5

(c) Describe the transformations that could be applied to the graph of $f(x) = x^2$ in order to obtain the graph of $g(x)$.

reflect over the x-axis

stretch vertically by a factor of 3

move down 5 units

shift left 3 units.

Free Response Questions: Show your work!

14. You are in the market for a new cell phone and begin shopping around.

(a) You have a 20% off coupon from the manufacturer for the purchase. Find a function f that models the purchase price as a function of the sticker price x after applying the coupon.

$$f(x) = x - 0.2x$$
$$= 0.8x$$

(b) Your cell phone provider is also offering a \$10 discount on any new phone. Find a function g that models the purchase price as a function of the sticker price x after applying the discount only.

$$g(x) = x - 10$$

(c) On your first trip to the store, you speak with Eli. He tells you that you can take advantage of both deals. He will apply the discount and then apply the coupon to the reduced price. Find a function that models Eli's offer.

$$f(g(x)) = f(x-10)$$
$$= 0.8(x-10)$$
$$= 0.8x - 8$$

(d) On your second trip to the store, you talk to Abe. He also says that you can take advantage of both deals, but he tells you that he will apply the coupon and then the discount. Find a function that models Abe's offer.

$$g(f(x)) = g(0.8x)$$
$$= 0.8x - 10$$

(e) From which salesperson should you buy the cellphone? Abe

Free Response Questions: Show your work!

15. Given the following polynomial, determine whether each of the statements are either **TRUE** or **FALSE**. Write your answer on the line preceding each statement.

$$P(x) = (x+5)(x-2)^3(x+6)^2(x-7)$$

(a) False The graph will cross the x-axis at $x = -6$.

The graph will touch the x-axis at $x=-6$ since the multiplicity of $(x+6)$ is even.

(b) True For $y = P(x)$, $y \to \infty$ as $x \to \infty$.

The leading term of $P(x)$ is x^7, so the end behavior is as $x \to -\infty$, $y \to -\infty$

as $x \to \infty$, $y \to \infty$.

(c) False The graph will touch the x-axis, but not cross, at $x = 2$.

The graph will cross at $x=2$ since the multiplicity of $(x-2)$ is odd.

(d) False For $y = P(x)$, $y \to \infty$ as $x \to -\infty$.

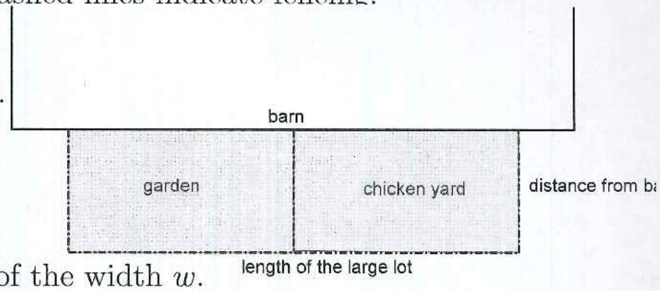
Same as (b).

(e) False The graph will have at most 3 local extrema.

The degree of $P(x)$ is 7, so the graph can have at most 6 local extrema.

Free Response Questions: Show your work!

16. Robert has a 195 foot roll of chicken wire fencing with which he would like to enclose a garden plot and a chicken yard alongside his barn. He wants to use all of his chicken wire while creating the maximum area possible for his garden and chickens. He plans to use the barn as one side of his lot with a common fence between the garden and the chicken yard, as shown in the figure, where the dashed lines indicate fencing.



Let w be the width of the lot, the distance from the barn.

Let l be the length of the lot, parallel to the barn.

(a) Write an equation for the length l in terms of the width w.

$$l + 3w = 195$$

$$l = 195 - 3w$$

(b) Write an equation for the area of the lot in terms of the width w.

$$A = l \cdot w$$
$$= (195 - 3w)\,w$$
$$= -3w^2 + 195w$$

(c) Use any method to determine how wide he should build the lot in order to maximize the total area.

$$\frac{-b}{2a} = \frac{-195}{-6} = 32.5 \text{ ft.}$$

(d) What will the length of the maximum area lot be?

$$l = 195 - 3w = 195 - 3(32.5)$$
$$= 97.5 \text{ ft.}$$

(e) What is the maximum area?

$$A = l \cdot w = (97.5)(32.5) = 3168.75 \text{ ft}^2$$

Free Response Questions: Show your work!

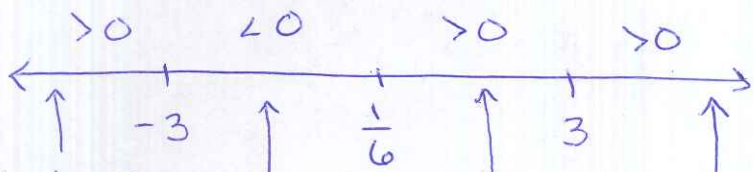
17. Solve the following inequality **algebraically**. SHOW YOUR WORK!!! Give your answer in **interval notation**.

$$\frac{(6x-1)(x+3)}{x^2-6x+9} \geq 0$$

$$\frac{(6x-1)(x+3)}{(x-3)(x-3)} \geq 0$$

x-intercepts: $\frac{1}{6}$, -3

vertical asymptotes: $x = 3$



test -5

when $x=-5$,

we have

$$\frac{(-31)(-2)}{(-8)(-8)} > 0$$

test 1

when $x=1$

we have

$$\frac{(5)(4)}{(-2)(-2)} > 0$$

test 0

when $x=0$

we have

$$\frac{(-1)(3)}{(-3)(-3)} < 0$$

test 5

when $x=5$

we have

$$\frac{(29)(8)}{(2)(2)} > 0$$

$$(-\infty,-3] \cup \underbrace{[\tfrac{1}{6},3) \cup (3,\infty)}$$

we don't include 3 since it is the vertical asymptote.