Multiple Choice Questions

1. Take a deep breath and write "I got this ...".

2. Expand the following logarithmic expression

$$\ln\left(\frac{(x+8)(x-4)}{(x-2)^2}\right)^{3/5}, \quad x>4.$$

A. $\frac{3}{5}\ln(x+8)+\frac{3}{5}\ln(x-4)-\frac{6}{5}\ln(x-2)$

B. $3\ln(x+8)-5\ln(x-4)-\frac{6}{5}\ln(x-2)$

C. $\frac{3}{5}\ln(x^2+12x-32)-\frac{6}{5}\ln(x-2)$

D. $\ln(x+8)+\ln(x-4)+\ln 36\ln(x-2)-\ln 5$

E. $\frac{3}{5}\ln(x+8)+\frac{3}{5}\ln(x-4)-\frac{3}{5}\ln(x-2)$

$\frac{3}{5}\ln\left(\frac{(x+8)(x-4)}{(x-2)^2}\right)$

$= \frac{3}{5}\left[\ln(x+8)+\ln(x-4)-\ln(x-2)^2\right]$

$= \frac{3}{5}\left[\ln(x+8)+\ln(x-4)-2\ln(x-2)\right]$

3. Express as a single logarithm:

$$3\log_a x-\frac{5}{3}\log_a y+\frac{1}{6}\log_a w-5\log_a z.$$

A. $\log_a \frac{x^3z^5}{w^{1/6}y^{5/3}}$

B. $\log_a \frac{x^3w^{1/6}}{y^{5/3}z^5}$

C. $\log_a \frac{x^3y^{5/3}}{w^{1/6}z^5}$

D. $\log_a \frac{x^3}{w^{1/6}y^{5/3}z^5}$

E. $\log_a\left(3x-\frac{5}{3}y+\frac{1}{6}w-5z\right)$

$= \log_a x^3 - \log_a y^{5/3} + \log_a w^{1/6} - \log_a z^5$

$= \log_a\left(\frac{x^3 w^{1/6}}{y^{5/3}z^5}\right)$

4. Solve the following equation for x.

$$\log(x+9) = 1 - \log x$$

base = 10

$\log(10) = 1 - \log(1)$
$1 = 1 - 0$ ✓

A. $\{-10, 1\}$
B. $\{1\}$
C. $\{-1\}$
D. $\{-1, 10\}$
E. No solution

$\log(x+9) + \log(x) = 1$

$\log((x)(x+9)) = 1$

$\log(x^2 + 9x) = 1$

$x^2 + 9x = 10^1$

$x^2 + 9x - 10 = 0$

$(x+10)(x-1) = 0$

$x = -10 \quad x = 1$

check answers $x = -10$ not solution!
$x > 0$ solutions

5. Describe how to transform the graph of the basic function $g(x) = \log x$ into the graph of the function $f(x) = 9\log(4-x)$. $= 9\log(-(x-4))$

A. Reflect across the y-axis, translate 4 units to the right, and vertically stretch by a factor of 9.
B. Reflect across the y-axis, translate 4 units to the left, and vertically stretch by a factor of 9.
C. Reflect across the x-axis, translate 4 units to the left, and vertically stretch by a factor of 9.
D. Reflect across the x-axis, translate 4 units to the right, and vertically stretch by a factor of 9.
E. Reflect across the y-axis, translate 4 units to the right, and vertically compress by a factor of 9.

6. Convert $162°$ to radians.

A. $\frac{9\pi}{10}$

B. π

C. $\frac{4\pi}{5}$

D. $\frac{9\pi}{5}$

E. $\frac{4\pi}{10}$

$$\frac{162^\circ}{360^\circ} = \frac{x}{2\pi}$$

$$x = \frac{162}{360}(2\pi)$$

or

$$\frac{162}{180}(\pi) = \frac{9}{10}\pi$$

7. Use basic trigonometric identities to simplify

$$\frac{\cos^2 x}{\sin^2 x} + \cos x \sec x.$$

A. $\csc x$

B. $\sec^2 x$

C. $\tan x$

D. $\cot^2 x$

E. $\csc^2 x$

$$= \frac{1-\sin^2(x)}{\sin^2(x)} + \cos(x)\left(\frac{1}{\cos(x)}\right)$$

$$= \frac{1}{\sin^2(x)} - \frac{\sin^2(x)}{\sin^2(x)} + 1$$

$$= \frac{1}{\sin^2(x)} - 1 + 1$$

$$= \frac{1}{\sin^2(x)} = \csc^2(x)$$

(There are many other ways to do this problem)

8. Solve the following equation for x:

$$9^{2x} \cdot 27^{3-x} = \frac{1}{9}$$

A. $\{-10\}$

B. $\{-11\}$

C. $\left\{\frac{9+\sqrt{87}}{6}, \frac{9-\sqrt{87}}{6}\right\}$

D. $\{-8\}$

E. $\{10\}$

$$(3^2)^{2x}(3^3)^{3-x} = 3^{-2}$$

$$3^{4x}\,3^{9-3x} = 3^{-2}$$

$$3^{4x+9-3x} = 3^{-2}$$

$$4x+9-3x = -2$$

$$x = -11$$

9. Find $\log_3 13$.

A. 2.334718

B. 2.801661

C. 2.334318

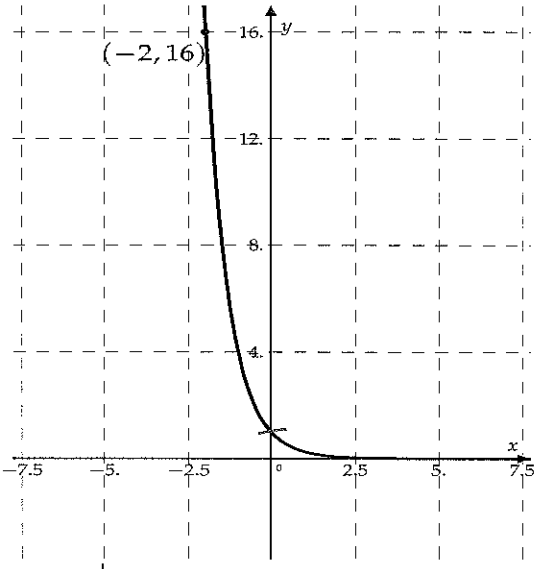
D. 0.428317

E. 3.344718

$$\log_3(13) = \frac{\log(13)}{\log(3)}$$

$$\log_3(13) = \frac{\ln(13)}{\ln(3)}$$

10. Find the exponential function whose graph is given



A. $y = -4^x$

B. $y = 4^{x+1}$

C. $y = 4^{-x}$

D. $y = x^4$

E. $y = 4^x$

must be C.

check

$4^{-(-2)} = 4^2 = 16$

$\therefore (-2, 16)$ on graph 4^{-x}

11. A 50-gallon barrel is filled completely with pure water. Salt water with a concentration of 0.5 lb/gal is then pumped into the barrel, and the resulting mixture overflows at the same rate. The amount of salt in the barrel at time t is given by

$$Q(t) = 25\left(1 - e^{-0.08t}\right)$$

where t is measured in minutes and $Q(t)$ is measured in pounds. How much salt is in the barrel after 10 min?

A. 15.9 pounds

B. 15.8 pounds

C. 14.8 pounds

D. 13.8 pounds

E. 13.7 pounds

$Q(10) = 25\left(1 - e^{-0.08(10)}\right)$

$= 25\left(1 - e^{-.8}\right) = 13.766$

Free Response Questions

12. Suppose you're driving your car on a cold winter day (18° F outside) and the engine overheats (at about 220° F). When you park, the engine begins to cool down. The temperature H of the engine t minutes after you park satisfies the equation

$$\ln\left(\frac{H-18}{202}\right) = -0.13t.$$

(a) Solve the equation for H.

$H = 202e^{-0.13t} + 18$

$\frac{H-18}{202} = e^{-0.13t}$

$H - 18 = 202e^{-0.13t}$

(b) Use part (a) to find the temperature of the engine after 25 min ($t = 25$). Round your answer to two decimal places.

$H = 202e^{-0.13(25)} + 18 = 25.83$

13. Given that the terminal point for angle θ is $\left(\frac{5}{8}, -\frac{7}{8}\right)$, find

(a) $\sin\theta = \frac{-7}{\sqrt{74}}$

(b) $\cos\theta = \frac{5}{\sqrt{74}}$

(c) $\tan\theta = \frac{-7}{5}$

(d) $\cot\theta = \frac{-5}{7}$

(e) $\sec\theta = \frac{\sqrt{74}}{5}$

(f) $\csc\theta = -\frac{\sqrt{74}}{7}$

↑ not on unit circle so divide by distance (h)

$h = \sqrt{\frac{25}{64} + \frac{49}{64}} = \sqrt{\frac{74}{64}} = \frac{\sqrt{74}}{8}$

$\left(\frac{\frac{5}{8}}{\frac{\sqrt{74}}{8}}, \frac{-\frac{7}{8}}{\frac{\sqrt{74}}{8}}\right)$

$= \left(\frac{5}{\sqrt{74}}, -\frac{7}{\sqrt{74}}\right)$ on unit circle

14. Given that $\sec\theta = 7$ and $\sin\theta < 0$, find the values of the other trigonometric functions of θ.

$\sec\theta = 7$

$\frac{1}{\cos\theta} = 7$

$\cos\theta = \frac{1}{7}$

$\sin^2(\theta) = \pm\sqrt{1-\left(\frac{1}{7}\right)^2}$

$= \pm\sqrt{\frac{48}{49}} = \frac{\pm\sqrt{48}}{7}$

(a) $\sin\theta$

$-\frac{\sqrt{48}}{7}$

(b) $\cos\theta$

$\frac{1}{7}$

(c) $\tan\theta$

$-\sqrt{48}$

(d) $\cot\theta$

$\frac{-1}{\sqrt{48}}$

(e) $\sec\theta$

7

(f) $\csc\theta$

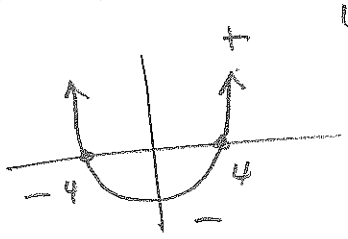
$\frac{-7}{\sqrt{48}}$

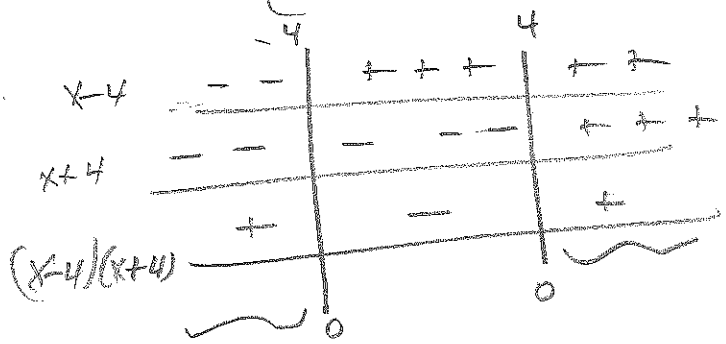
15. Find the domain of the function $f(x) = \ln(x^2 - 16)$.

$x^2 - 16 > 0$

$(x-4)(x+4) > 0$

use graph or chart





$(-\infty, -4) \cup (4, \infty)$

16. Vilfredo Pareto (1848–1923) observed that most of the wealth of a country is owned by a few members of the population. Pareto's Principle is

$$\log P = \log c - k \log W,$$

where W is the wealth level (how much money a person has) and P is the number of people in the population having that much money.

(a) Solve the equation for P.

$\log P = \log c - \log W^k = \log \frac{c}{W^k}$

$P = \frac{c}{W^k}$

(b) Assume $k = 2.5$, $c = 7{,}000$, and W is measured in millions of dollars. Use part (a) to find the number of people who have \$2 million or more. Round the answer to the nearest integer.

NOT ON THIS EXAM

(c) How many people have \$11 million or more? Again round the answer to the nearest integer.

17. The elk population in a certain region is given by the function $E(t) = 1050 + 150 \sin\left(\frac{4t}{5}\right)$ where the time t is measured in years.

(a) What is the largest number of elk present in the region at any time?

max of $\sin\left(\frac{4t}{5}\right) = 1$

$\Rightarrow \quad 1050 + 150(1) = 1{,}200$

(b) How much time elapses between occurrences of the largest and smallest elk population?

what is period of E?

$0 < \frac{4t}{5} < 2\pi$

$0 < 4t < 10\pi$

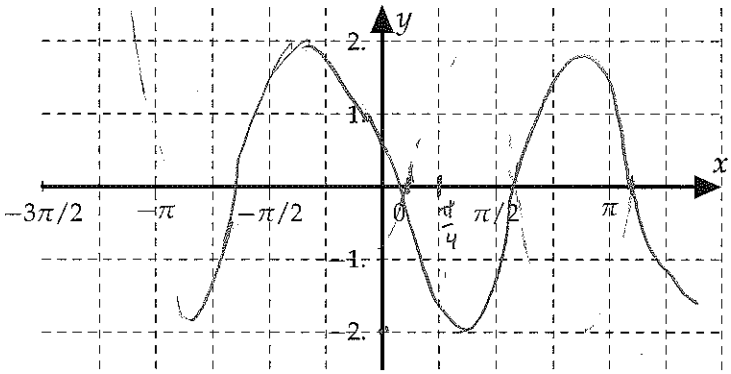
$0 < t < \frac{10\pi}{4}$

period $= \frac{5\pi}{2}$

largest to smallest $= \frac{1}{2}$ period

$= \frac{5\pi}{4}$

18. Find the amplitude, period and phase shift of $-2\sin\left(2x-\frac{\pi}{4}\right)$. Sketch a graph of the function on the supplied axes.



stretch by 2
flip

$0 \le 2x - \frac{\pi}{4} \le 2\pi$

$\frac{\pi}{4} \le 2x \le 2\pi + \frac{\pi}{4}$

$\frac{\pi}{8} \le x \le \pi + \frac{\pi}{8}$

period $= \pi + \frac{\pi}{8} - \frac{\pi}{8} = \pi$

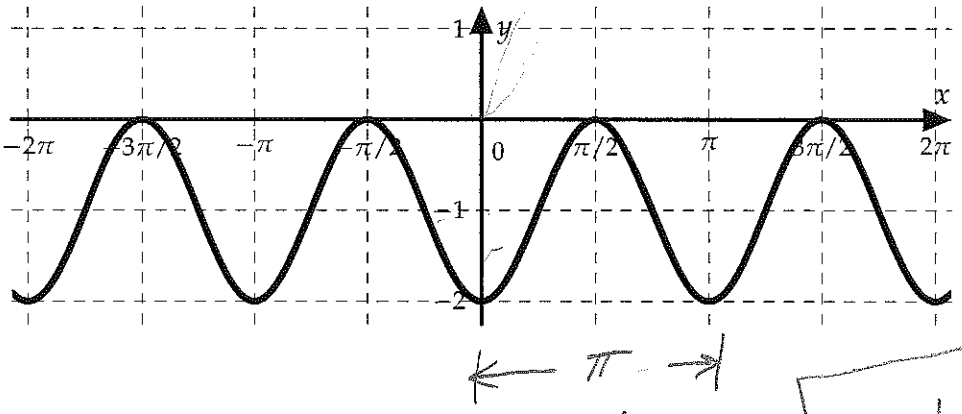
phase shift $= \frac{\pi}{8}$ to right

amplitude $= 2$

19. Simplify the expression $\dfrac{\sin^2 x - \cos^2 x}{1 - \tan^2 x}$.

NOT ON Exam

20. Determine the equation of the function that is graphed below:

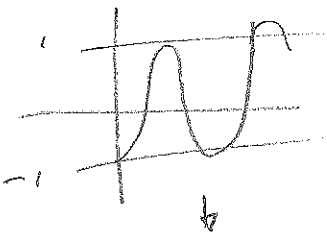


shift down 1

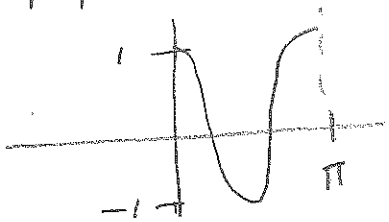
π

period (shrink by 2)

$\Rightarrow$ $-\cos(2t)-1$



flip



21. Use the properties of logarithms to expand the expression

$$\log\sqrt[6]{x\sqrt[6]{y\sqrt[6]{z}}}$$

in a form with no logarithm of a product, quotient, or power.

$\log\left(x\left(y\,z^{1/6}\right)^{1/6}\right)^{1/6}$

$= \frac{1}{6}\log\left(x\left(y\,z^{1/6}\right)^{1/6}\right)$

$\frac{1}{6}\left(\log(x) + \frac{1}{6}\log\left(y\,z^{1/6}\right)\right) = \frac{1}{6}\left(\log x + \frac{1}{6}\left(\log y + \frac{1}{6}\log z\right)\right)$

$= \frac{1}{6}\log x + \frac{1}{36}\log y + \frac{1}{216}\log z$

or

$= \log x^{1/6} y^{1/36} z^{1/216}$

$= \log x^{1/6} + \log y^{1/36} + \log z^{1/216}$

same

END OF TEST