MA 110 Algebra and Trigonometry for Calculus
Fall 2016
Exam 3
Tuesday, November 15

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: __________

This exam has twelve multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

EXAMPLE	A	B	C	D	E
Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
13		10
14		10
15		10
16		10
17		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Suppose that $\ln(x) = 2$ and $\ln(y) = 3$, find $\ln\left(\dfrac{x^2}{y}\right)$.

   (A) 7
   (B) -1
   (C) 5
   (D) 6
   (E) 1

   E.

   We have $\ln(\frac{x^2}{y}) = 2\ln(x) - \ln(y) = 2 \cdot 2 - 3 = 1$.

2. Simplify $\ln(\ln(e^{e^e}))$

   (A) e
   (B) 1
   (C) e^e
   (D) 0
   (E) Undefined.

   A.

   $\ln(\ln(e^{e^e})) = \ln(e^e) = e$

3. Solve the equation
$$\ln(3x + 4) = \ln(x + 1) + \ln(2)$$

   (A) -2
   (B) -1
   (C) 0
   (D) 1
   (E) There is no solution

   E.

   Using properties of logarithms, the equation becomes $\ln(3x+4) = \ln(2x+2)$. Applying the exponential function gives $3x + 4 = 2x + 2$, which simplifies to $x + 2 = 0$, and finally $x = -2$. However, if we substitute $x = -2$, we find that $\ln(3 \cdot -2 + 4) = \ln(-2)$ and $\ln(-2 + 1) = \ln(-1)$ are undefined.

Record the correct answer to the following problems on the front page of this exam.

4. Solve the equation $4^x = A$ with $A > 0$.

   (A) $x = \dfrac{\ln(A)}{\ln(4)}$

   (B) $x = \dfrac{A}{4}$

   (C) $x = \dfrac{A}{\ln(4)}$

   (D) $x = 4A$

   (E) $x = \dfrac{\ln(A)}{4}$

   A. If we apply the function ln to both sides, we have $\ln(4^x) = \ln(A)$. Thus $x\ln(4) = \ln(A)$ or $x = \ln(A)/\ln(4)$.

5. Find a model of the form $f(t) = A2^{-t/b}$ so that $f(0) = 16$ and $f(3) = 8$

   (A) $A = 8$ and $b = 3$

   (B) $A = 16$ and $b = 1/3$

   (C) $A = 16$ and $b = 3$

   (D) $A = 16$ and $b = -3$

   (E) $A = 8$ and $b = -3$

   C. Setting $t = 0$, we have $f(0) = 16 = A$, thus $A = 16$. Next, since $f(3) = 8 = 16\cdot 2^{-3/b}$, we must have $2^{-3/b} = 1/2$ or $b = 3$.

6. List the transformations needed to transform the graph of $y = \sin(x)$ to the graph of $y = 4 - 3\sin(x)$.

   (A) Reflect the graph in x-axis, stretch vertically by a factor of 3, translate down by 4 units.

   (B) Reflect the graph in x-axis, stretch vertically by a factor of 3, translate up by 4 units.

   (C) Reflect the graph in y-axis, stretch vertically by a factor of 3, translate to the right by 4 units.

   (D) Reflect the graph in y-axis, shrink vertically by a factor of 3, translate to the right by 4 units.

   (E) Reflect the graph in x-axis, shrink vertically by a factor of 3, translate up by 4 units.

   B. Reflecting the graph of $y = \sin(x)$ in the x-axis gives the graph of $y = -\sin(x)$, vertically stretching by a factor of 3 gives the graph of $y = -3\sin(x)$, and then translating up by three units gives the graph $y = 4 - 3\sin(x)$.

Record the correct answer to the following problems on the front page of this exam.

7. Find the radian measure of an angle if the terminal side is obtained by rotating the initial side through 1/5 of a circle.

   (A) $\frac{2\pi}{5}$
   (B) $\frac{1}{5}$
   (C) $\frac{2}{5}$
   (D) 72
   (E) 72π

   A.

   One rotation is 2π radians so 1/5 of a rotation is $2\pi/5$ radians.

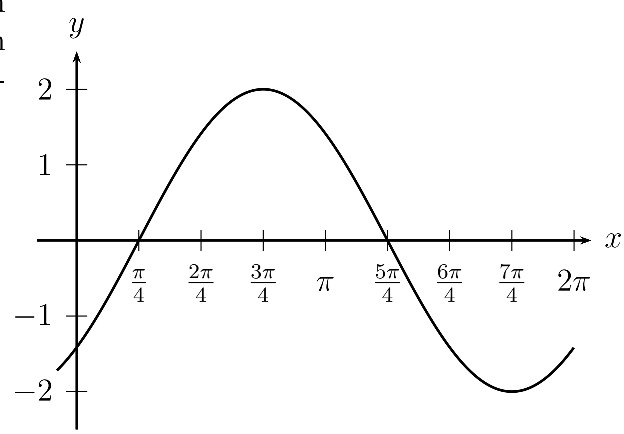
8. The point $(24, -7)$ lies on the terminal side of an angle in standard position of radian measure t. Find $\cos(t)$ and $\tan(t)$.

   (A) $\cos(t) = \frac{24}{25}$ and $\tan(t) = -\frac{7}{24}$
   (B) $\cos(t) = \frac{24}{25}$ and $\tan(t) = -\frac{24}{7}$
   (C) $\cos(t) = -\frac{7}{25}$ and $\tan(t) = -\frac{7}{24}$
   (D) $\cos(t) = -\frac{7}{24}$ and $\tan(t) = -\frac{7}{25}$
   (E) $\cos(t) = \frac{24}{25}$ and $\tan(t) = -\frac{7}{25}$

   A.

   The point $(24, -7)$ is $r = \sqrt{24^2 + (-7)^2} = \sqrt{625} = 25$ units from the origin. We have $\cos(t) = x/r = 24/25$ and $\tan(t) = y/x = -7/24$. Thus the answer is B.

9. To the right is the graph of a function $f(x) = A\sin(x+B)$ which passes through the points $(\frac{\pi}{4}, 0)$ and $(\frac{3\pi}{4}, 2)$. Give the values of A and B.



   (A) $A = -2$ and $B = -\frac{\pi}{4}$
   (B) $A = 2$ and $B = \frac{\pi}{4}$
   (C) $A = -2$ and $B = \frac{\pi}{4}$
   (D) $A = 2$ and $B = -\frac{\pi}{4}$
   (E) None of the other answers are correct.

   D.

   The sin wave begins at $\frac{\pi}{4}$. Thus, $\frac{-B}{1} = \frac{\pi}{4}$ or $B = -\frac{\pi}{4}$. The wave reaches a maximum distance of 2 from the $x-$axis and is not reflected in the $x-$axis. Therefore, $A = 2$, and the desired function is $f(x) = 2\sin(x - \pi/4)$.

Record the correct answer to the following problems on the front page of this exam.

10. Suppose that $\sin(x) = \dfrac{4}{5}$ and $\cos(x) < 0$, find $\tan(x)$.

   (A) $\dfrac{3}{4}$

   (B) $-\dfrac{3}{4}$

   (C) $\dfrac{4}{3}$

   (D) $-\dfrac{4}{3}$

   (E) Undefined

   D.

   We have $\cos(x) = -\sqrt{1-(4/5)^2} = -3/5$. Then $\tan(x) = \sin(x)/\cos(x) = -4/3$.

11. Suppose that an angle of t radians is in standard position. The terminal side of the angle lies on the line $y = 4x$ and has $x < 0$. Find $\tan(t)$.

   (A) $\dfrac{1}{4}$

   (B) $-\dfrac{1}{4}$

   (C) 4

   (D) -4

   (E) Undefined.

   C.

   If we let $x = -1$, then $y = -4$ and $(-1, -4)$ gives a point on the line. We have $\tan(t) = y/x = 4$.

12. How many solutions of the equation $\sin(x) = .7$ are there in the interval $[6\pi, 8\pi]$?

   (A) 4

   (B) 1

   (C) 0

   (D) 2

   (E) 3

   D.

   The equation will have two solutions.

Free Response Questions: Show your work!

13. A population grows exponentially and triples in 4 years. Suppose that there are 111 individuals on 1 January 1950.

    (a) Determine the population in 4 years.

$3 \cdot 111 = 333$	1 point

    (b) Find P and k so that the population t years after 1 January 1950 is given by $f(t) = Pe^{kt}$.

    Give an exact value of k using a logarithm function and a decimal approximation that is correctly rounded to four decimal places.

We want $f(0) = P = 111$.	1 point
To find k we use that $f(4) = Pe^{k\cdot 4} = 3P$. Thus, we want $e^{4k} = 3$.	2 points
We apply the natural logarithm to solve this equation and obtain $4k\ln(e) = \ln(3)$. Or $k = \ln(3)/4 \approx 0.2747$.	2 points for k, 1 point for decimal approximation.

    (c) Find the population on 1 January 1955. Round your answer to the nearest whole number.

$111 \cdot e^{5\cdot\ln(3)/4} \approx 438.25$ or 438 after rounding.	1 point.

    (d) During which year does the population reach 1000?

We need to solve $111e^{T\ln(3)/4} = 1000$	1 point
to obtain $T = \ln(1000/111) \cdot 4/\ln(3) \approx 8.0036$.	1 point
Thus, we reach 1000 individuals in the beginning of 1958.	1 point for year.

Free Response Questions: Show your work!

14. Let $V(t)$ denote the volume of air in an adult's lung at time t seconds. We measure volume in cubic centimeters. Assume that $V(t)$ is given by the formula

$$V(t) = 400 + 75\sin\left(\frac{\pi t}{3}\right).$$

(a) How much air is in the lung after 15 seconds?

$V(15) = 400 + 75\sin(5\pi) = 400\text{cm}^3$.	2 points

(b) Give the maximum and minimum volume of air in the lungs.

Maximum is 475 cm^3	3
Minimum is 325 cm^3	2

(c) How many breaths per minute does the person take? Assume that each breath corresponds to one period of the function V.

Each breath takes one period of the function $\sin(\pi t/3)$ or 6 seconds. Thus, there are 10 breaths in a minute.	3 points

Free Response Questions: Show your work!

Addition formula for cosine

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

15. Suppose that $\cos(x) = \dfrac{\sqrt{5}}{5}$ and $\sin(x) > 0$ and that $\sin(y) = \dfrac{2\sqrt{5}}{5}$ and $\cos(y) < 0$.

(a) Find $\sin(x)$ and $\cos(y)$.

Since $\cos^2(x) + \sin^2(x) = 1$,	Using Pythagorean identity, 1 point.
then $\left(\frac{\sqrt{5}}{5}\right)^2 + \sin^2(x) = 1$.	
Hence, $\sin^2(x) = 1 - \frac{1}{5}$	Correct arithmetic, 2 points.
or $\sin(x) = \frac{2\sqrt{5}}{5}$ since $\sin(x) > 0$	Correct choice, 1 points.
Similarly, we find $\cos(y) = -\frac{\sqrt{5}}{5}$	Correct answer, 2 points.

(b) Use the addition formula for the cosine function to find $\cos(x+y)$.

$\cos(x+y)$	
$= \cos(x)\cos(y) - \sin(x)\sin(y)$	
$= (\frac{\sqrt{5}}{5})(-\frac{\sqrt{5}}{5}) - (\frac{2\sqrt{5}}{5})(\frac{2\sqrt{5}}{5})$	Correct substitutions, 2 points.
$= -\frac{5}{25} - \frac{20}{25} = -1$.	Correct answer, 2 points.

16. Simplify the expression

$$\cos^2(x)\Big(1+\tan^2(x)\Big)-\frac{\sin^2(x)}{1+\cos(x)}$$

to obtain a single trigonometric function. Show your work.

$\cos^2(x)(1+\tan^2(x))-\frac{\sin^2(x)}{1+\cos(x)}$	
$=\cos^2(x)+\cos^2(x)\frac{\sin^2(x)}{\cos^2(x)}-\frac{\sin^2(x)}{1+\cos(x)}$	Definition of tangent 2 points, distributive law 1 point,
$=\cos^2(x)+\cos^2(x)\frac{\sin^2(x)}{\cos^2(x)}-\frac{1-\cos^2(x)}{1+\cos(x)}$	rewrite $\sin^2(x)$ as $1-\cos^2(x)$ 2 points,
$=\cos^2(x)+\sin^2(x)-\frac{(1-\cos(x))(1+\cos(x))}{1+\cos(x)}$	factor $1-\cos^2(x)$, 1 point.
$=1-(1-\cos(x))$	Pythagorean identity, 1 point. Cancel in fraction 1 point.
$=\cos(x)$	Answer 2 points.
ALTERNATELY	
$\cos^2(x)(1+\tan^2(x))-\frac{\sin^2(x)}{1+\cos(x)}$	
$=\cos^2(x)\big(\sec^2(x)\big)-\frac{\sin^2(x)}{1+\cos(x)}$	Pythagorean identity 2 points.
$=\cos^2(x)\sec^2(x)-\frac{1-\cos^2(x)}{1+\cos(x)}$	Pythagorean identity 2 points.
$=\cos^2(x)\dfrac{1}{\cos^2(x)}-\frac{1-\cos^2(x)}{1+\cos(x)}$	Definition of secant 1 points.
$=1-\frac{(1-\cos(x))(1+\cos(x))}{1+\cos(x)}$	simplifying fraction 1 points, factoring numerator 1 point.
$=1-(1-\cos(x))$	simplifying fraction 1 points.
$=\cos(x)$	Answer 2 points.

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}, \sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$$

17. (a) Give the exact value of $\cos(\frac{5\pi}{4})$.

From the unit circle, $\cos(5\pi/4) = -\sqrt{2}/2$	2 points

(b) Use the half-angle identity to find an exact value for $\cos\left(\frac{5\pi}{8}\right)$ and simplify the result. Hint: Your answer will include radicals, but should not involve any trigonometric functions.

$\cos(\frac{5\pi}{8}) = -\sqrt{\frac{1+(-\sqrt{2}/2)}{2}}$	3 points, deduct one point for incorrect sign
Choose negative square root since $5\pi/8$ is in the second quadrant	
$= -\sqrt{\frac{2-\sqrt{2}}{4}}$	Accept this
$= -\frac{\sqrt{2-\sqrt{2}}}{2}$	or this 2 points

(c) Use a calculator to compute a decimal approximation of the expression in part b), correctly rounded to three decimal places.

$-\sqrt{\frac{2-\sqrt{2}}{4}} \approx -0.383$	No credit, if the decimal does not correspond the answer in part b). 2 points

(d) Use a calculator to compute a decimal approximation to $\cos\left(\frac{5\pi}{8}\right)$, correctly rounded to three decimal places. Hint: You may use parts c) and d) to check your answer to part b).

-0.383	1 point