

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Last 4 digits of student ID #: \_\_\_\_\_

This exam has eleven multiple choice questions (five points each) and five free response questions (nine points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

**On the multiple choice problems:**

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

**On the free response problems:**

1. Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

**Multiple Choice Answers**

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E

← +5 points on curve

**Exam Scores**

Question	Score	Total
MC		55
12		9
13		9
14		9
15		9
16		9
Score		100

Record the correct answer to the following problem on the front page of this exam.

- (1) If a radioactive substance decays with a decay constant of  $r = 0.0005$ , how long to the nearest year until there is one third of the substance remaining.

- A) 2197 years.
- B) 5364 years.
- C) The time can not be determine with out the initial quantity.
- D) There will never be one third left.
- E) 148 years.

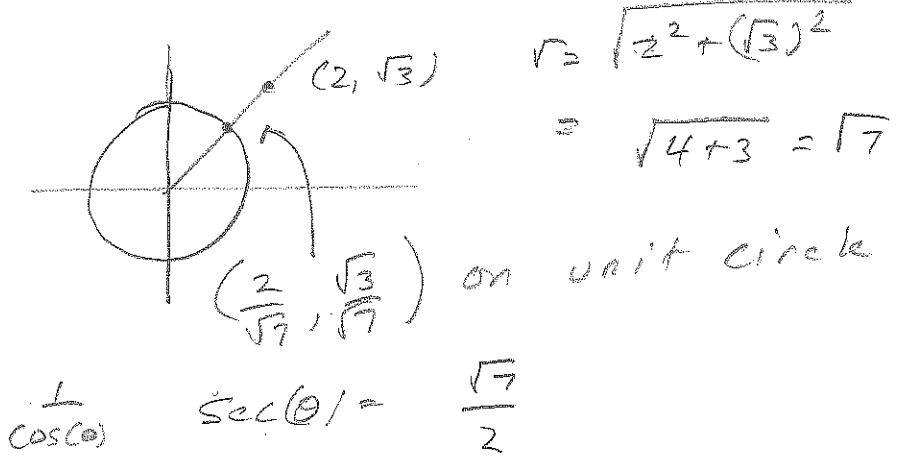
$$\frac{Q_0}{3} = Q_0 e^{-0.0005t}$$

$$\ln\left(\frac{1}{3}\right) = t$$

$$-0.0005$$

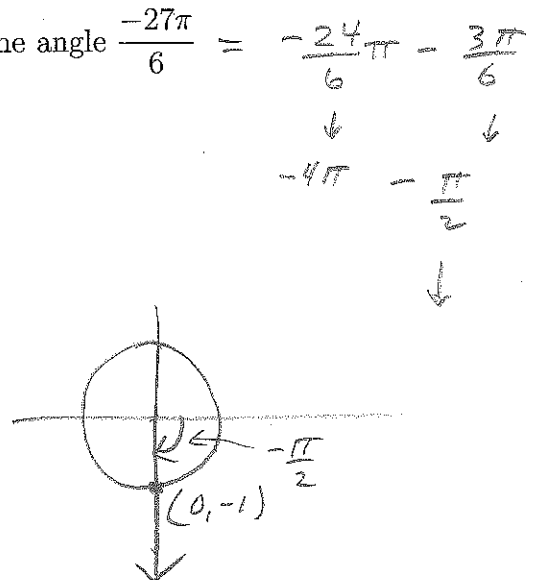
- (2) Find  $\sec(\theta)$  for the angle  $\theta$  whose terminal ray contains the point  $(2, \sqrt{3})$ .

- A)  $\frac{1}{2}$
- B)  $\frac{2}{\sqrt{7}}$
- C)  $\frac{\sqrt{7}}{\sqrt{3}}$
- D)  $\frac{\sqrt{7}}{2}$
- E)  $\frac{2}{\sqrt{3}}$



- (3) Find the point on the unit circle corresponding to the angle  $\frac{-27\pi}{6} = \frac{-24\pi}{6} - \frac{3\pi}{6}$

- A)  $\left(\frac{\sqrt{-3}}{2}, \frac{1}{2}\right)$
- B)  $(0, -1)$
- C)  $\left(\frac{\sqrt{1}}{2}, \frac{\sqrt{3}}{2}\right)$
- D)  $(1, 0)$
- E)  $\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$



Record the correct answer to the following problem on the front page of this exam.

(4) Find  $\sin(t)$  if  $\cos(t) = \frac{-1}{5}$  and  $0 < t < \pi$ .

- A)  $\frac{4}{5}$
- B)  $\frac{-\sqrt{24}}{5}$
- C)  $\frac{-\sqrt{2}}{3}$
- D)  $\frac{\sqrt{5}}{5}$
- E)  $\frac{2\sqrt{6}}{5}$**

↓  
QII

$$\begin{aligned} \sin(t) &= \pm \sqrt{1 - \left(\frac{-1}{5}\right)^2} \\ &= \pm \sqrt{1 - \frac{1}{25}} \\ &= \pm \sqrt{\frac{24}{25}} = \pm \frac{\sqrt{24}}{5} \end{aligned}$$

→  $\sin(t) > 0$

$$\sin(t) = \frac{\sqrt{24}}{5} = \frac{2\sqrt{6}}{5}$$

(5) Solve the equation completely.

- A)  $\frac{\pi}{4} \pm 2n\pi$  for  $n$  any integer.
- B)  $\frac{4}{\pi}$  only.
- C) The equation has no solution**
- D)  $\frac{\pi}{4}$  only.
- E)  $\frac{4}{\pi} \pm 2n\pi$  for  $n$  any integer.

$$\sec(t) = \frac{\sqrt{2}}{2}$$

$$\frac{1}{\cos(t)} = \frac{\sqrt{2}}{2}$$

$$\cos(t) = \frac{2}{\sqrt{2}} > 1$$

No solution

(6) Find the period of the graph of

$$f(t) = \tan\left(2x - \frac{\pi}{4}\right)$$

- A)  $\frac{\pi}{2}$**
- B)  $\frac{\pi}{8}$
- C)  $2\pi$
- D)  $\frac{\pi}{4}$
- E)  $\pi$

$$0 < 2x - \frac{\pi}{4} < \pi$$

$$\frac{\pi}{4} < 2x < \pi + \frac{\pi}{4}$$

$$\frac{\pi}{8} < x < \frac{\pi}{2} + \frac{\pi}{8}$$

distance =  $\frac{\pi}{2}$

Record the correct answer to the following problem on the front page of this exam.

(7) Which one of the following expression is equivalent to  $\sec^2(x) - \tan^2(x) - \sin^2(x)$ .

A)  $\cos^2(x)$

B)  $\csc^2(x)$

C)  $1 - \cos(x)$

D)  $\cos(x) \sin(x)$

E) None of the above are equivalent to the expression given.

$$= \frac{1}{\cos^2(x)} - \frac{\sin^2(x)}{\cos^2(x)} - \sin^2(x)$$

$$= \frac{1 - \sin^2(x)}{\cos^2(x)} - \sin^2(x)$$

$$\frac{\cos^2(x) - \sin^2(x)}{\cos^2(x)}$$

$$1 - \sin^2(x) = \cos^2(x)$$

(8) Find an angle in radian measure equivalent  $-240^\circ$  angle.

A)  $\frac{4\pi}{3}$

B)  $\frac{2\pi}{3}$

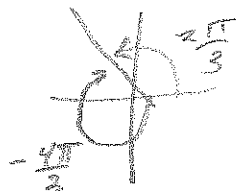
C)  $\frac{\pi}{3}$

D)  $\frac{-\pi}{3}$

E)  $\frac{3\pi}{4}$

$$\frac{-240^\circ}{180^\circ} \pi$$

$$= -\frac{4\pi}{3}$$



(9) How far does a point of intersection of a terminal ray with the unit circle travel if the angle of the ray starts at 0 and rotates through an angle of  $135^\circ$ ?

A) Half way around the circle.

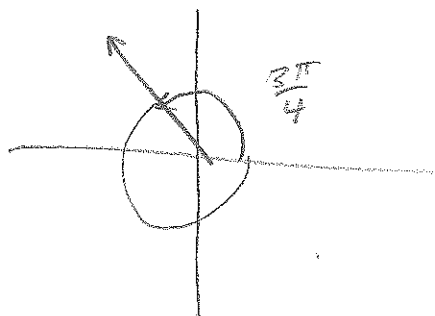
B)  $\pi$

C)  $135^\circ$

D)  $\frac{3\pi}{4}$

E) The distance is unknown since the radius of the circle is not given.

$$135^\circ = \frac{3\pi}{4}$$



Record the correct answer to the following problem on the front page of this exam.

(10) Which one of the following statements is true for all values of  $x$ ?

- A)  $\cos(-x) = \cos(x)$  True
- B)  $\sin(x + \pi) = \sin(x)$  False
- C)  $\tan(-x) = \tan(x)$  False
- D)  $\sec(x) = \csc(x - \pi)$  False
- E) None of the above are true. False

(11) Which one of the statements is false?

- T A)  $\sin(x) \sec(x) - \tan(x) = 0$
- T B)  $\frac{\tan(x)}{\cot(x)} = \tan^2(x) = \tan^2(x) = \tan^2(x)$
- T C)  $\sec^2 x + \csc^2(x) = \sec^2(x) \csc^2(x)$
- T D)  $\cot^2(x) = \csc^2(x) - 1$
- T E)  $\sin(x)(\csc^2(x) - \cot^2(x)) = \sin(x)$

$$\frac{\sin(x)}{\cos(x)} - \tan(x) = 0$$

$$\frac{1}{\cos^2(x)} + \frac{1}{\sin^2(x)} = \frac{\sin^2 + \cos^2}{\cos^2 \sin^2}$$

$$= \frac{1}{\cos^2 \sin^2}$$

$$\rightarrow \frac{\cos^2}{\sin^2} = \frac{1}{\sin^2} - 1 \Rightarrow \cot^2(x) = \csc^2(x) - 1$$

Useful formulas

$$\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\sin(x - y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

All are true everyone got this (curve)

Free Response Questions: Show your work!

(12) Suppose  $\sin(\theta) = \frac{-2}{5}$  and  $\theta$  is in the third quadrant.

(a) Find  $\cos(\theta)$

↓  
 $\cos(\theta) < 0$

$$\cos(\theta) = -\sqrt{1 - \sin^2(\theta)} = -\sqrt{1 - \left(\frac{-2}{5}\right)^2} = -\sqrt{1 - \frac{4}{25}}$$

(b) Find  $\cos(2\theta)$

$$= -\sqrt{\frac{21}{25}} = -\frac{\sqrt{21}}{5}$$

$$\cos(2\theta) = \cos(\theta + \theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$$

$$= \left(\frac{-\sqrt{21}}{5}\right)\left(\frac{-\sqrt{21}}{5}\right) - \left(\frac{-2}{5}\right)\left(\frac{-2}{5}\right)$$

$$\frac{21}{25} - \frac{4}{25} = \frac{17}{25}$$

Free Response Questions: Show your work!

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(13) Use the sum or difference of angles formulas to find exactly  $\cos\left(\frac{\pi}{12}\right)$

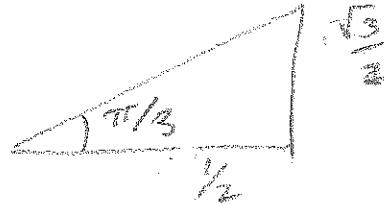
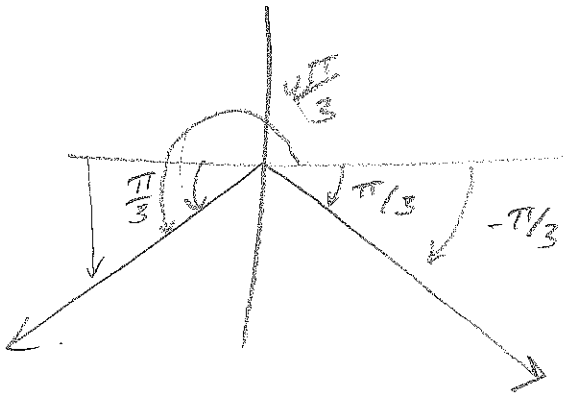
Hint: Write  $\frac{\pi}{12}$  as the sum or difference of two angles from the reference triangles.

$$\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$$

$$\begin{aligned}\cos\left(\frac{\pi}{12}\right) &= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

Free Response Questions: Show your work!

(14) Find all the solutions to  $\sin(t) = \frac{-\sqrt{3}}{2}$  ← Q III, Q IV



$$t = \frac{4\pi}{3}, \quad t = -\frac{\pi}{3}$$

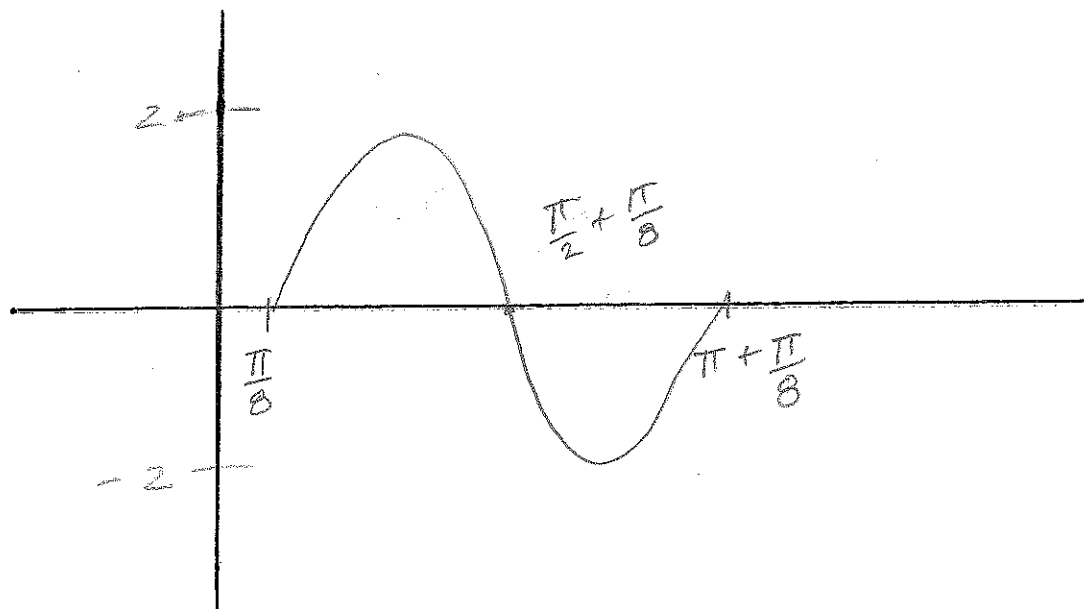
$$\frac{4\pi}{3} \pm 2n\pi, \quad -\frac{\pi}{3} \pm 2n\pi$$

$n = 0, 1, 2, 3, \dots$



Free Response Questions: Show your work!

- (15) Graph one period of  $f(x) = 2 \cos\left(2x - \frac{\pi}{4}\right)$ . Label the  $x$  and  $y$  axis accurately including the  $x$  intercepts.



$$0 < 2x - \frac{\pi}{4} < 2\pi$$

$$\frac{\pi}{2} < 2x < 2\pi + \frac{\pi}{4}$$

$$\frac{\pi}{8} < x < \pi + \frac{\pi}{8}$$



shift

$$\leftarrow \pi \rightarrow$$

Free Response Questions: Show your work!

(16) Solve the equation for  $x$

$$\log_3(x) = -\log_3(x-6) + 3.$$

$$\log_3(x) + \log_3(x-6) = 3$$

$$\log_3(x(x-6)) = 3$$

$$x^2 - 6x = 27$$

$$x^2 - 6x - 27 = 0$$

$$(x-9)(x+3) = 0$$

$x = 9$     $x = -3$  ← not a solution

Check

$$\log_3(9) = -\log_3(9-6) + 3$$

$$2 = -\log_3(3) + 3$$

$$2 = -1 + 3 \quad \checkmark$$

$$\log_3(-3) = -\log_3(-3-6) + 3$$

↑  
undefined  
↑