MA 110 Algebra and Trigonometry for Calculus — Spring 2017
Exam 3 — Tuesday, 11 April 2017

Name: Key

Section: ______

Last 4 digits of student ID #: ______

This exam has twelve multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

EXAMPLE	A	B	C	D	E
Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		50
13		10
14		10
15		10
16		10
17		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. If you invest \$5000 into an account that is compounded continuously at an annual rate of 3.2%, how much is in the account after 5 years and 5 months?

   (A) \$5,962.19

   (B) \$28,297.44

   (C) \$7,458.08

   (D) \$5,946.31

   (E) \$27,964.02

$A = Pe^{rt}$ $P = 5000$ $r = .032$

$t = 5 + \frac{5}{12} = 5.41\overline{6}$

$A = 5000e^{(.032)(5.4166)} = \$5946.31$

2. Between 1850 and 1920, the population P of the United States (in millions) in the year t was given by $P(t) = 23.1788(1.0302^t)$, where t is the number of years since 1850. Find the population in the year 1907.

   (A) $\approx 352,137,389$ people

   (B) $\approx 126,362,743$ people

   (C) $\approx 790,527,622$ people

   (D) $\approx 285,457,703$ people

   (E) $\approx 442,677,507$ people

$t = 1907 - 1850 = 57$

$P(57) = 23.1788(1.0302^{57}) =$

126.3627428 million

$126.3627428 * 1,000,000 =$

126362742.8 people

3. Find the domain of $f(x) = \ln(x^2 + 3x - 4)$.

   (A) $(-\infty, -1) \cup (4, \infty)$

   (B) $(1, \infty)$

   (C) $(-4, 1)$

   (D) $(4, \infty)$

   (E) $(-\infty, -4) \cup (1, \infty)$

$x^2 + 3x - 4 > 0$

$(x+4)(x-1) > 0$

$+ \quad - \quad +$

$-4 \quad 1$

$(-\infty, -4) \cup (1, \infty)$

Record the correct answer to the following problems on the front page of this exam.

4. Suppose that $\log(s) = -3$ and $\log(t) = 1$, find

$$\log\left(\frac{t^2}{s^3}\right) =$$

$\log t^2 - \log s^3$

$2\log t - 3\log s$

$2(1) - 3(-3)$

$2 + 9$

11

(A) -8

(B) -7

(C) 11

(D) -26

(E) 28

5. Solve.

$$\log_3(x-4) + \log_3(x-6) = 1$$

$\log_3[(x-4)(x-6)] = 1$

$x^2 - 10x + 24 = 3^1$

$x^2 - 10x + 21 = 0$

$(x-7)(x-3) = 0$

$x - 7 = 0$ $\quad x - 3 = 0$

$x = 7$ $\quad x = 3$ extraneous

makes negative

(A) $x = 3$ and $x = 7$

(B) $x = 7$

(C) $x = 5$ and $x = 8$

(D) $x = 8$

(E) $x = 6$

6. Find the radian measure of a *standard position* angle between 0 and 2π that is coterminal with $-\frac{5\pi}{6}$.

$-\frac{5\pi}{6} + 2\pi$

$-\frac{5\pi}{6} + \frac{12\pi}{6}$

$\frac{7\pi}{6}$

(A) $\frac{7\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $-\frac{7\pi}{6}$

(D) $\frac{5\pi}{6}$

(E) $-\frac{5\pi}{6}$

Record the correct answer to the following problems on the front page of this exam.

7. Find the point P on the unit circle corresponding to the angle $\frac{7\pi}{3}$.

(A) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(B) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(C) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(D) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

(E) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\frac{7\pi}{3}$ more than 2π

$\frac{7\pi}{3} - 2\pi = \frac{7\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

8. The point $(12, -5)$ lies on the terminal side of an angle in *standard position* measuring t radians. Find $\sin(t)$ and $\tan(t)$.

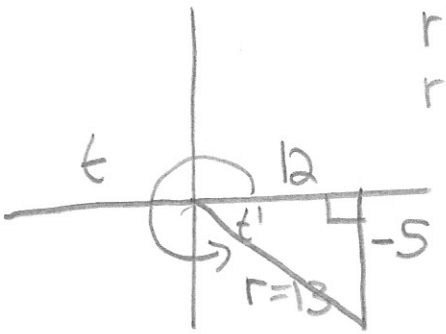
(A) $\sin(t) = \frac{5}{13}$ and $\tan(t) = -\frac{12}{5}$

(B) $\sin(t) = \frac{12}{13}$ and $\tan(t) = -\frac{5}{12}$

(C) $\sin(t) = -\frac{5}{13}$ and $\tan(t) = -\frac{5}{12}$

(D) $\sin(t) = -\frac{12}{13}$ and $\tan(t) = -\frac{5}{12}$

(E) $\sin(t) = -\frac{12}{5}$ and $\tan(t) = -\frac{12}{13}$



$r^2 = 12^2 + (-5)^2$

$r^2 = 144 + 25$

$r^2 = 169$

$r = 13$

$\sin t = \frac{y}{r} = \frac{-5}{13}$

$\tan t = \frac{y}{x} = \frac{-5}{12}$

9. How many solutions of the equation $\cos(t) = -.7$ are there in the interval $[-\pi, 2\pi]$?

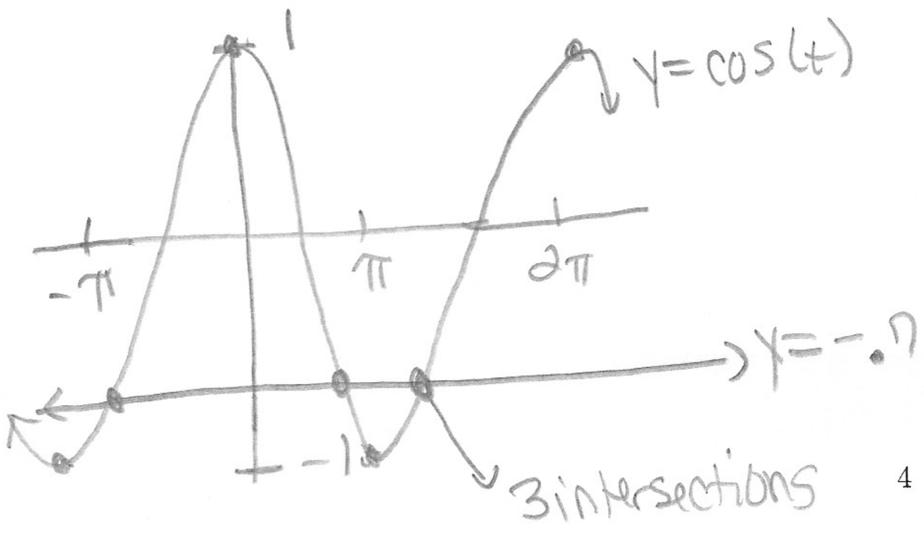
(A) 3

(B) 4

(C) 1

(D) 0

(E) 2



Record the correct answer to the following problems on the front page of this exam.

10. State the amplitude, period, and phase shift of the function.

$$h(t) = -2\sin\left(3t + \frac{\pi}{6}\right) = -2\sin\left[3\left(t+\frac{\pi}{18}\right)\right]$$

(A) Amplitude: -2 Period: $\frac{2\pi}{3}$ Phase Shift: $-\frac{\pi}{18}$

(B) Amplitude: -2 Period: $\frac{2\pi}{3}$ Phase Shift: $-\frac{\pi}{2}$

(C) Amplitude: 2 Period: $\frac{2\pi}{3}$ Phase Shift: $-\frac{\pi}{2}$

(D) Amplitude: 2 Period: $\frac{2\pi}{3}$ Phase Shift: $-\frac{\pi}{18}$

(E) Amplitude: 2 Period: $\frac{\pi}{3}$ Phase Shift: $-\frac{\pi}{2}$

Ampl $= |-2| = 2$

period $= \frac{2\pi}{3}$

phase shift $\frac{\pi}{18}$ left

11. Suppose that $\tan(x) = -\sqrt{3}$ and $\sin(x) > 0$, find $\cos(x)$.

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) $-\frac{\sqrt{3}}{2}$

(E) $-\frac{1}{2}$

If tan negative x in QII or Q IV

If sin positive x in Q I or QII

X must be in Q II so cos is negative

$\tan^2 x + 1 = \sec^2 x$

$(-\sqrt{3})^2 + 1 = \sec^2 x$

$3 + 1 = \sec^2 x$

$4 = \sec^2 x$

$\cos^2 x = \frac{1}{4}$

$\cos x = \sqrt{\frac{1}{4}} =$

$-\frac{1}{2}$

12. Simplify.

$$\frac{\tan(\theta)}{\sin(\theta)} =$$

(A) $\csc(\theta) - \sin(\theta)$

(B) $\sec^2(\theta)$

(C) $\sec(\theta)$

(D) $\sec(\theta) - \cos(\theta)$

(E) $\sin(\theta) - \cos(\theta)$

$\tan\theta \cdot \frac{1}{\sin\theta} =$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta} = \frac{1}{\cos\theta} =$

$\sec\theta$

Free Response Questions: Show your work!

13. Consider the function

$$f(x) = \log(7x - 5).$$

(a) Find the domain of f. Give your answer in interval notation.

$7x-5>0$

$7x>5$

$x>\frac{5}{7}$

$\boxed{\left(\frac{5}{7},\infty\right)}$

(b) Find the range of f. Give your answer in interval notation.

$\boxed{(-\infty,\infty)}$ same as Domain of $f^{-1}(x)$

(c) Find $f^{-1}(x)$

$x=\log(7y-5)$

$10^x=7y-5$

$10^x+5=7y$

$\frac{10^x+5}{7}=y$

$\boxed{f^{-1}(x)=\frac{10^x+5}{7}}$

(d) Find the domain of f^{-1}. Give your answer in interval notation.

$(-\infty,\infty)$

(e) Find the range of f^{-1}. Give your answer in interval notation.

$\left(\frac{5}{7},\infty\right)$ same as domain of $f(x)$

14. Suppose you're driving your car on a cold winter day (20° F outside) and the engine overheats (at about 220° F). When you park, the engine begins to cool down. The temperature U of the engine t minutes after you park satisfies the equation

$$\ln\left(\frac{U-20}{200}\right) = -0.11t.$$

(a) Why is this equation not accurate for temperatures below 20° F outside?

$\frac{U-20}{200}$ must be greater than 0 so $U > 20$; cannot take ln of non-positive values.

(b) Solve the equation for U.

$\ln\left(\frac{U-20}{200}\right) = -.11t$ change to exponential form

$e^{-.11t} = \frac{U-20}{200}$ mult. by 200

$200e^{-.11t} = U-20$ add 20

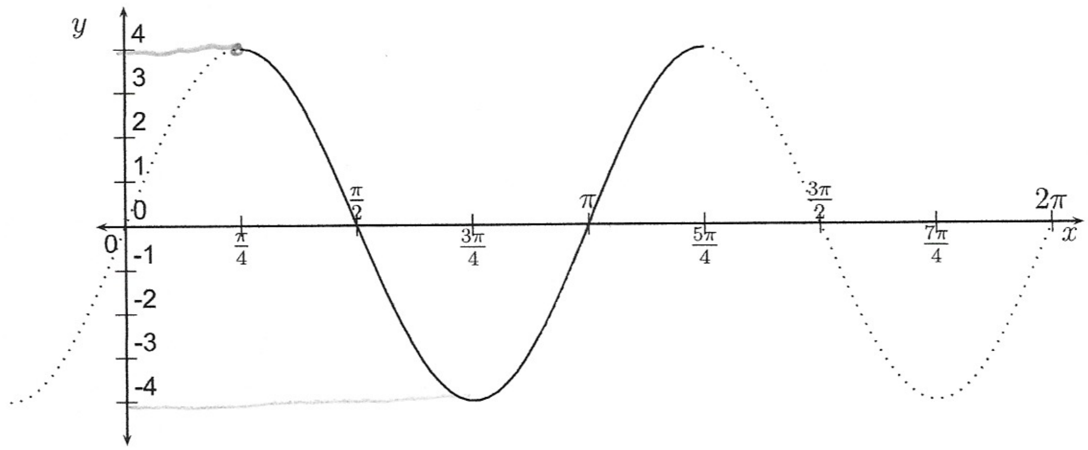
$\boxed{200e^{-.11t} + 20 = U}$

(c) Use part (b) to find the temperature of the engine after 20 min ($t = 20$).

$U = 200e^{-.11(20)} + 20 =$

$22.1606 + 20 = \boxed{42.1606°F}$

15. Consider the following graph of the form $g(t) = A\cos(bt + c)$



(a) Find the amplitude, period, and phase shift from the graph.

Amplitude 4

Period π $\rightarrow \frac{5\pi}{4} - \frac{\pi}{4} = \pi$

Phase Shift $\pi/4$ right

(b) Find $A, b,$ and c and state the rule of the function.

$\boxed{A=4}$ $\frac{2\pi}{b} = \pi \rightarrow \boxed{b=2}$

$g(t) = 4\cos[2(t-\frac{\pi}{4})] =$

$4\cos(2t - \frac{\pi}{2})$

$\boxed{C = -\frac{\pi}{2}}$

$g(t) =$ $4\cos(2t - \frac{\pi}{2})$

(c) Describe the graph transformations needed to produce the graph of $g(t)$ from the graph of $f(t) = \cos(t)$. [NOTE: The order of your transformations matters!!!]

1.) vert. stretch by 4
2.) horizontal shrink by a factor of 2
3.) horizontal shift right $\frac{\pi}{2}$

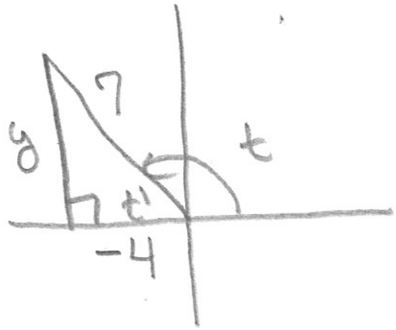
Free Response Questions: Show your work!

16. Find the values of each of the trigonometric functions at t under the given conditions.

$$\cos(t) = -\frac{4 \to x}{7 \to r} \quad \text{and} \quad \cot t < 0$$

$\cos t < 0$ Q II + Q III   must be Q II

$\cot t < 0$ Q II + Q IV



$7^2 = (-4)^2 + y^2$

$49 = 16 + y^2$

$33 = y^2$

$\sqrt{33} = y$

$\sin(t) = \frac{y}{r} = \frac{\sqrt{33}}{7}$

$\tan(t) = \frac{y}{x} = \frac{\sqrt{33}}{-4}$

(a) $\sin(t) =$ $\frac{\sqrt{33}}{7}$.

(b) $\tan(t) =$ $-\frac{\sqrt{33}}{4}$.

(c) $\csc(t) =$ $\frac{7}{\sqrt{33}} = \frac{7\sqrt{33}}{33}$.

(d) $\sec(t) =$ $-\frac{7}{4}$.

(e) ~~tan~~ cot$(t) =$ $-\frac{4}{\sqrt{33}} = \frac{-4\sqrt{33}}{33}$.

Free Response Questions: Show your work!

17. Simplify the expression

$$\sin^2(x)\Big(1+\cot^2(x)\Big) - \frac{\cos^2(x)}{1+\sin(x)}$$

to obtain a single trigonometric function. SHOW YOUR WORK!

$$\sin^2 x\,(1+\cot^2(x)) - \frac{\cos^2 x}{1+\sin x}$$

$$\sin^2 x\,(\csc^2 x) - \frac{(1-\sin^2 x)}{1+\sin x}$$

$$\cancel{\sin^2 x}\left(\frac{1}{\cancel{\sin^2 x}}\right) - \frac{(\cancel{1+\sin x})(1-\sin x)}{\cancel{1+\sin x}}$$

$$1 - \frac{(1-\sin x)}{1}$$

$$1 - 1 + \sin x$$

$$\boxed{\sin x}$$