MA 110 Algebra and Trigonometry for Calculus — Fall 2016
Exam 4 — 12 December 2016

Name: ______________________________

Section: ______________________________

Last 4 digits of student ID #: ________

This exam has twelve multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

EXAMPLE	**A** (shaded)	B	C	D	E
Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

Scores

Question	Score	Total
MC		50
13		10
14		10
15		10
16		10
17		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

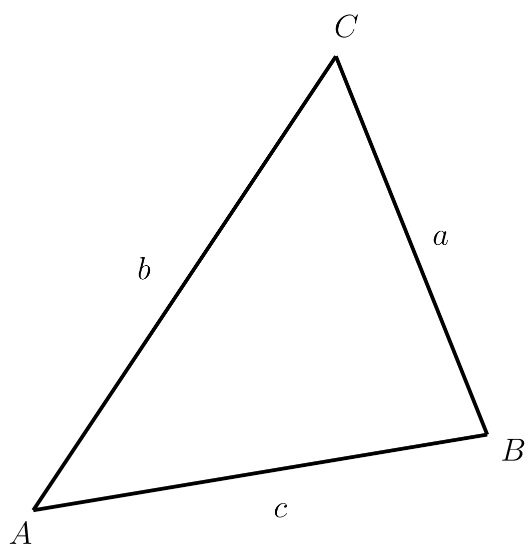
The following formulae may be useful. All assume our standard naming convention for triangles where the side of length a is opposite the angle of measure A, the side of length b is opposite the angle of measure B and the side of length c is opposite the angle of measure C.

Law of sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C).$$



Record the correct answer to the following problems on the front page of this exam.

1. The area of a rectangle whose sides are s and t is $A = s \cdot t$. If we know that the sum of s and t is 16, find a function that gives the area in terms of t.

   (A) $A(t) = 16t$

   (B) $A(t) = 16 - t^2$

   (C) $A(t) = 16t - t^2$

   (D) $A(t) = 8t - t^2$

   (E) $A(t) = t^2 - 8t$

   C.

   We have $s + t = 16$ so that $s = 16 - t$. Substituting this into $A = s \cdot t$, gives $A(t) = 16t - t^2$.

2. Consider the equation $7y + 51x = 123$.

   (A) This equation defines y as a function of x, but does not define x as a function of y.

   (B) This equation defines x as a function of y, but does not define y as a function of x.

   (C) This equation does not define y as a function of x and does not define x as a function of y.

   (D) This equation defines x a function of y and defines y as a function of x.

   (E) None of the above options is correct.

   D.

   In order to determine that y is a functions of x, we determine that for every input of x, you would get exactly one output for y. Similarly, you would also determine that x is a function of y.

3. Find the remainder when the polynomial $P(x) = x^3 - 10x + 2$ is divided by $x + 3$.

   (A) 1

   (B) 2

   (C) 3

   (D) 4

   (E) 5

   E.

   The remainder can be found by evaluating the polynomial $P(x) = x^3 - 10x + 2$ at $x = -3$. Thus the remainder is $P(-3) = -27 + 30 + 2 = 5$.

Record the correct answer to the following problems on the front page of this exam.

4. If $3,500 is deposited into a bank account that compounds continuously with an annual rate of 4%, how much will the account be worth in 5 years?

   (A) $\$3,500 \cdot e^{0.2}$

   (B) $\$3,500 \cdot (1.04)^5$

   (C) $\$3,500 \cdot (1.02)^{10}$

   (D) $\$3,500 \cdot e^{20}$

   (E) None of the above.

   A.

   Note that answer B is for interest compounded annually and answer C is for interest compounded semi-annually.

5. Describe how to obtain the graph of $y = 2 + \sin(x + 3)$ from the graph of $y = \sin(x)$.

   (A) Shift 3 units to the right and 2 units up.

   (B) Shift 2 units to the left and 3 units up.

   (C) Shift 3 units to the left and 2 units up.

   (D) Shift 2 units to the right and 2 units down.

   (E) Shift 3 units to the right and 2 units down.

   C.

   The 3 affects the graph horizontally by shifting left, and the 2 affects the graph vertically by shifting up.

6. How many solutions does the equation $\sin(x) = 0.3$ have in the interval $[-\pi, \frac{\pi}{2}]$? (The angle x is measured in radians.)

   (A) 0

   (B) 1

   (C) 2

   (D) 3

   (E) 4

   B.

   The interval restrictions in the question include quadrants I, III, and IV. Only one of those quadrants produces positive outputs for the sine function.

Record the correct answer to the following problems on the front page of this exam.

7. Assume that $\cos(u) = 3/5$ and $-\pi/2 \leq u \leq 0$. Find $\sin(u)$. (The angle u is measured in radians.)

   (A) $3/5$
   (B) $4/5$
   (C) $-3/5$
   (D) $-4/5$
   (E) $-4/3$

   D. From the Pythagorean identity, we have $\sin(u) = \pm\sqrt{1 - 9/25} = \pm 4/5$. Since the terminal side of the angle u is in the fourth quadrant, we have $\sin(u) < 0$.

8. A model plane is 20 meters above the ground and is flying away from an observer located on the ground. Find the value of the angle of elevation θ when the distance x from the observer to the plane is 170 meters.

   A.

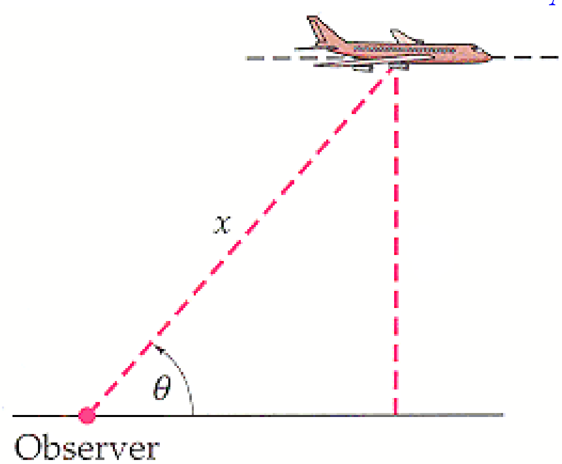
   (A) $\theta = \sin^{-1}(2/17)$

   (B) $\theta = \cos^{-1}(2/17)$

   (C) $\theta = \tan^{-1}(2/17)$

   (D) $\theta = \tan^{-1}(17/2)$

   (E) $\theta = \sin^{-1}(17/2)$



   We have $\sin(\theta) = 20/170$ and thus $\theta = \sin^{-1}(2/17)$.

9. Two sides of a triangle have length 8 and 10 units and the angle between them measures $60°$. Find the length of the third side.

   (A) $\sqrt{83}$
   (B) $\sqrt{84}$
   (C) $\sqrt{85}$
   (D) $\sqrt{86}$
   (E) $\sqrt{87}$

   B.

   From the law of cosines the third side has length c satisfying

$$\begin{aligned} c^2 &= 10^2 + 8^2 - 2 \cdot 8 \cdot 10 \cos(60°) \\ &= 100 + 64 - 80 = 84. \end{aligned}$$

Record the correct answer to the following problems on the front page of this exam.

10. A line is given in parametric form by $x = 2t + 1$ and $y = 3t + 2$. Eliminate the parameter t to find an equation x and y which defines the line. Put your answer into the form $y = mx + b$.

    (A) $y = \frac{2}{3}x - \frac{1}{2}$

    (B) $y = \frac{2}{3}x + 2$

    (C) $y = \frac{2}{3}x + \frac{4}{3}$

    (D) $y = \frac{3}{2}x + 1$

    (E) $y = \frac{3}{2}x + \frac{1}{2}$

    E. Solving the first parametric equation for t, we have $t = \frac{x-1}{2}$. Substituting this expression for t into the second parametric equation, we have $y = 3(\frac{x-1}{2}) + 2$ and simplifying we get $y = \frac{3}{2}x - \frac{3}{2} + 2 = \frac{3}{2}x + \frac{1}{2}$

11. Suppose that an angle of t radians is in standard position. The terminal side of the angle lies on the line $y = 4x$ and has $x < 0$. Find $\tan(t)$.

    (A) $1/4$

    (B) $-1/4$

    (C) 4

    (D) -4

    (E) Undefined.

    C.

    If we let $x = -1$, then $y = -4$ and $(-1, -4)$ gives a point on the line. We have $\tan(t) = y/x = 4$.

12. Find a parametrization of the circle with center (21,13) and radius 6.

    (A) $x = 21 + 13\cos(t),\ y = 16 + 5\sin(t),\ 0 \le t \le 2\pi$

    (B) $x = 13 + 5\cos(t),\ y = 21 + 6\sin(t),\ 0 \le t \le \pi.$

    (C) $x = 21 + 6\cos(t),\ y = 13 + 6\sin(t),\ 0 \le t \le 2\pi$

    (D) $x = 13 + 6\cos(t),\ y = 13 + 6\sin(t),\ 0 \le t \le 2\pi$

    (E) $x = 21 + 6\sin(t),\ y = 21 + 6\cos(t),\ 0 \le t \le \pi$

    C. Beginning with the equation of the circle, $(x-21)^2 + (y-13)^2 = 6^2$, we can divide both sides by 6^2 and get $\left(\frac{x-21}{6}\right)^2 + \left(\frac{y-13}{6}\right)^2 = 1$. Now, recognizing the Pythagorean identity $\cos^2(t) + \sin^2(t) = 1$ provides the parameterization $\cos(t) = \frac{x-21}{6}$ and $x = 21 + 6\cos(t)$, while $\sin(t) = \frac{y-13}{6}$ and $y = 13 + 6\sin(t)$.

13. Consider the function

$$f(x) = \sqrt{\frac{4-x}{1+x}}.$$

Assume that the domain and the range are subsets of the real numbers.

(a) If possible, compute the following: $f(0)$ and $f(6)$. If one or more of the values do not exist (as real numbers) explain why.

$f(0) = \sqrt{\frac{4-0}{1+0}} = \sqrt{4} = 2$	1 point
$f(6) = \sqrt{\frac{4-6}{1+6}} = \sqrt{-2/7}$ which is not a real number because $-2/7 < 0$	1 point.
	Also give credit for the answer $i\sqrt{2/7}$.

(b) Find the domain of f and explain your reasoning.

Answer $(-1, 4]$	6 points
Can't take square root of negative number	1 point
Can't divide by zero	1 point

Free Response Questions: Show your work!

14. A ball is thrown up in the air from the roof of an 80 foot tall building. At time t seconds after it is thrown, the ball's height in feet above the ground is given by the the function $h(t) = 64t - 16t^2 + 80$.

(a) What is the height of the ball after 3 seconds?

$h(3) = 64 \cdot 3 - 16 \cdot 9 + 80 = 128$ m	2 point

(b) Find the time that the ball reaches its maximum height and give the maximum height.

$h(t) = -16(t-2)^2 + 144$ Vertex is $(2, 144)$ Maximum height is 144 m at time $t =$ 2 seconds	4 points

(c) At what time does the ball hit the ground?

Solve $h(t) = 0$	1 point
Solutions are -1,5	
Hits the ground after 5 seconds	3 points

Free Response Questions: Show your work!

15. A population grows exponentially and doubles in 5 years. We begin with 200 individuals on 1 January 2015.

(a) Determine the population on 1 January 2020.

Since the population doubles in 5 years, the population will be 400	1 point

(b) Find P and k so that the population t years after 1 January 2015 is given by $f(t) = Pe^{kt}$.

Give an exact value of k using a logarithm function and a decimal approximation that is correctly rounded to four decimal places.

We want $f(0) = P = 200$.	1 point
To find k we use that $f(5) = Pe^{k\cdot 5} = 2P$. Thus, we want $e^{5k} = 2$.	1 points
We apply the natural logarithm to solve this equation and obtain $5k\ln(e) = \ln(2)$. Or $k = \ln(2)/5 \approx 0.1386$.	2 points for k,
	1 point for decimal approximation.

(c) Find the population on 1 January 2023. Round your answer to the nearest whole number.

$200 \cdot e^{8\cdot\ln(2)/5} \approx 606.29$ or 606 after rounding.	1 point.

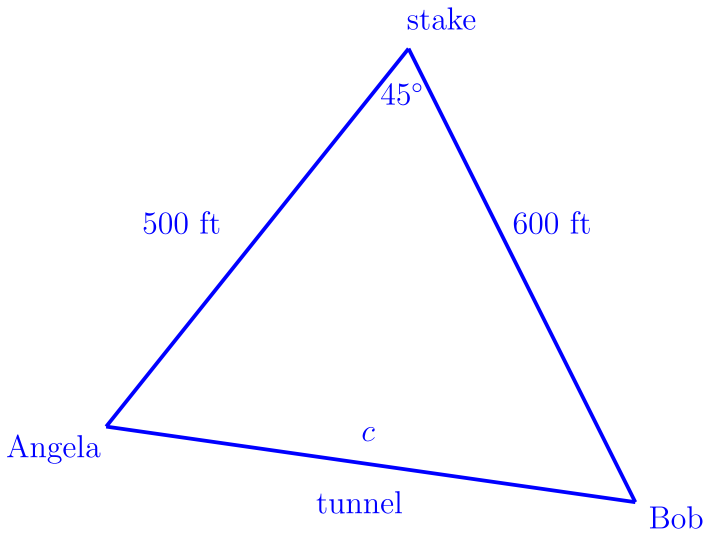
(d) During which year does the population reach 1200?

We need to solve $200e^{T\ln(2)/5} = 1200$	1 point
to obtain $T = \ln(5)\cdot 6/\ln(2) \approx 12.925$.	1 point
Thus, we reach 1200 individuals at the end of 2027.	1 point for year.

Free Response Questions: Show your work!

16. A straight tunnel is to be dug through a hill. Angela and Bob stand on opposite sides of the hill where the tunnel entrances are to be located. Both can see a stake located 500 meters from Angela and 600 meters from Bob. The angle formed by Angela, the stake, and Bob with vertex at the stake measures 45°.

    (a) Summarize the given information in a sketch. Your sketch should clearly label the triangle you use to answer part b).



4 points

    (b) Determine how long the tunnel must be? Round your answer to the nearest meter.

The Law of Cosines gives

$c^2 = 500^2 + 600^2 - 2 \cdot 500 \cdot 600 \cdot \cos(45°)$

$c = \sqrt{500^2 + 600^2 - 600,000\cos(45°)}$

$c \approx 431$ meters

6 points

Free Response Questions: Show your work!

17. Consider the ellipse given by the equation

$$x^2 + 4y^2 + 6x - 16y = 11.$$

(a) Give the center of the ellipse and the length of the major and minor axes.

Completing the squares give	
$x^2+6x+9+4(y^2-4y+4) = 11+9+16$	2 points
Dividing by 36 gives	
$\frac{(x+3)^2}{36} + \frac{(y-2)^2}{9} = 1$	1 point
Center is $(-3, 2)$	1 point
Axes of length $2(6) = 12$ and $2(3) = 6$	2 points

(b) Sketch the ellipse on the axes provided. | 4 points